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Measurement and Analysis of Subharmonics and Other Distortions in Compression Drivers

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ABSTRACT

The article presents the results of the subharmonic analysis of single suspension loudspeakers such as the compression drivers. The formation, synchronization and lock-in of the subharmonic frequencies, when working with the analytic form of the pressure signal, have been observed. A parametric or autoparametric (internal resonance) mechanism seems to be the main cause of these subharmonic excitations and responses. The modal shape of the moving assembly including the diaphragm, coil, former and suspension has an important role in the whole dynamic of these transducers.

One of the four analyzed samples presented a bilinear response, because of a spontaneous formation of sidebanding when it was excited in specific spectral regions. Another sample showed a combined behavior which was in between the bilinear character and the locked-in subharmonic. This sample exhibited regions in which the spontaneous sidebanding had a fractal structure. The basic control criterion of subharmonic responses is to avoid specific modes whose eigenfrequencies are multiples of each other.

1. INTRODUCTION

Subharmonics are spectral responses whose frequencies are sub-multiples of the forced frequencies, usually, their first sub-multiples. In 1831 M. Faraday reported observations of water waves responding at half the frequency of the mechanical excitation [1]. For many years, mathematicians have also given formal analytical explanations for this type of response [2]. In several engineering fields, the frequency analysis of dynamic

devices can exhibit these subharmonic spectral components. The topic has had importance in, among others, transportation systems and machinery dynamics.

The appearance of acoustic subharmonics radiated by loudspeakers was reported as early as 1935 by P.O. Pedersen [3] and, later by H. F. Olson [4]. P.O. Pedersen deduced that the subharmonic generation was caused by a resonance of the diaphragm. The author reported that some amount of time was needed for synchronization of

the electric signal with the subharmonic acoustic response. H.F.Olson gave importance to these distortion components, mainly in midrange speakers. He assigned a significant cause for this distortion to the suspension's nonlinearities and to the mechanical weakness of the cones. The work of W.J. Cunningham, published in 1951 [5], deals with subharmonics in loudspeakers of double suspension. The author gave specific explanations of the causes of the phenomenon with a closer approach to the cone modal behavior.

The subharmonic components deliver an unpleasant sound even at low amplitudes. Sometimes these produce abnormal sounds when speakers are swept with an oscillator. Moreover, when a tweeter is swept at a very high frequency range in which we no longer hear the signal, for it is beyond our sensitivity range, it is not unusual that we can suddenly hear a tone again.

As the appearance of distortion tones of subharmonic frequencies is not the only cause of distortion at frequencies lower than the excitation, we shall also see the spontaneous formation of sidebanding spectra, which is a nonlinear signature that occurs in some specific circumstances. Reference [6] is one of the most in-depth reports dealing with this topic in loudspeakers that refer both to subharmonics and to spontaneous sidebanding. The author does not give an explanation of the causes of these distortions. The paper's aim is to address the causes of these distortions in single suspension speakers.

2. PARAMETRIC EXCITED OSCILLATIONS

The main cause of subharmonic generation in dynamic systems is the parametric excitation. This excitation consists in the generation of forced vibrations by means of the periodic change of a parameter [7]. It is well known that children give energy to a swing by the periodic motion of their bodies, without any other external force. This single degree of freedom system, obtains the energy needed for the motion by means of the periodic change of the effective length of the swing. The natural frequency of the pendulum, for small oscillations (linear range) is only a function of its length and the acceleration of gravity, which is constant. The most common parametric excitation is the one exerted by a periodic parameter variation whose frequency is twice the natural frequency of the swing, but other relationships are possible too. For a mechanical system with low damping, the natural frequencies are functions of the inertias and stiffness of the constituent elements.

The mass seldom changes parametrically but in several cases stiffness can be modulated periodically.

One form of a parametric excited system is Mathieu's equation that derives from Hill's differential equation. The Mathieu differential equation has the expression:

$$d^2 x / dt^2 + \omega_0^2 [1 + A \cos(p^* t)] x = 0 \quad (1)$$

Where:

x : is the displacement

A : is the amplitude of the harmonic function.

p : is the circular frequency of the parameter forcing function, which is called pumping in some references.

ω_0 : is the natural circular frequency of the single degree of freedom system when it is free of any pumping action (A equals zero).

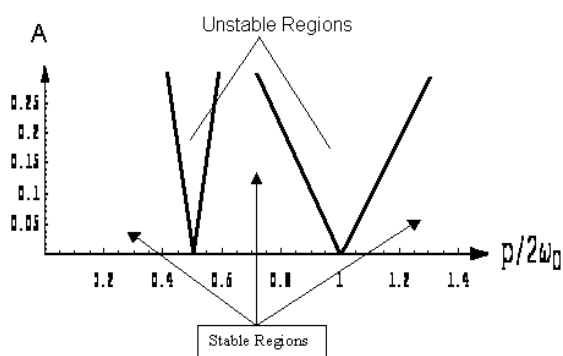


Figure 1: Two regions of instability of the Mathieu's parametric excited system. The unstable regions depart from points where the pumping circular frequencies (p) are twice or equals the natural circular frequency (ω_0) of the unperturbed system. The parameter A is the amplitude of the parametric excitation. When the amplitude A increases, the unstable region becomes broader.

Mathieu's equation has the simplification of not damping. If certain conditions between A and ω_0 are satisfied, there may exist periodic solutions of equation (1). Figure 1 depicts a simplified plane (A, ω_0) showing regions of stability and instability of the variable x of equation (1) depending on the relation of the pumping circular frequency in respect to the unperturbed natural circular frequency ($p / 2 \omega_0$), and also depending on the amplitude of the harmonic forced parametric function A . Notice that for the displayed normalized frequency range of Figure 1 the widest unstable region corresponds to the value of a pumping circular

frequency p which is double that of the natural circular frequency of the unperturbed system ω_0 . The region of coincidence of the pumping circular frequency and the natural circular frequency is unstable too, but it is narrower. An unlimited build up of amplitude takes place in a linear and conservative oscillating system under parametric excitation, which is called a parametric resonance. In practice, for real dissipative systems, the balance between the energy supplied to the system by the parametric action and the losses of the non conservative system gives an unstable or stable (bounded) output.

Mathieu's equation refers to conservative, linear parametric and single degree of freedom systems. Real systems are seldom linear and often have several degrees of freedom. A detailed explanation of the topic is beyond the aim of the paper. References [7] and [8] give detailed theories. From an engineering and experimental point of view there is an abundance of literature: reference [9] deals with the classic problem of the axial excitation of a bar. Reference [10] is concerned with the control of a vertical subharmonic motion by means of a pendular absorber. Reference [11] addresses the same topic but the parametric oscillator is a rotating pendulum and is controlled or absorbed by the vertical motion of its pivot. Professor P.O. Pedersen [3] refers to the subharmonic generated by loudspeakers as "Mathieu oscillations".

2.1. Internal Resonance and Combination Resonance

For real systems that have several degrees of freedom and a certain degree of nonlinearity, when they are excited, their time responses can contain large contributions from several modes. The specific trend of instability exhibited by some dynamic systems, whose natural frequencies are multiples or near multiples of other natural frequencies, has been known for many years [8]. The internal resonance condition is exhibited for those systems where two or more of its linear normal modes follow the condition expressed in (2).

$$f_p = n * f_q \quad (2)$$

Being: f_p and f_q any natural frequency of the system and n a positive integer, usually small.

When two natural frequencies of a system with quadratic nonlinearity have a ratio 2:1, there exists a saturation phenomenon that consists of the following: the system is excited at its highest frequency and it

responds to amplitude proportional to the excitation force (linear). If we increase the excitation force, a certain point is reached where the system loses its stability and both the lower and higher modes are excited. Those systems with these properties have a strong link between the modes and a high tendency to instability [12].

Modal interactions for systems with a certain level of nonlinearity can occur when the excitation frequency is close to the sum or the difference of two or more natural frequencies, but many other combinations may be given. This phenomenon of excitation of a nonlinear natural frequency, which can be a linear combination of some other normal modes, is called combination resonance [13]. The type of response when a combination resonance is excited depends on the type and amount of nonlinearity and the number of modes involved in the process. If a system is excited by a single sine signal of frequency f and has linear eigenfrequencies f_i , a general case of combination resonance can be obtained if the excitation frequency and some or all natural frequencies fulfill the following relationship:

$$f = \sum n_i * f_i \quad (3)$$

Being: n_i integers, positive or negative, usually small. In engineering there is an acceptance if the relation (3) is not exactly fulfilled and certain proximity is reached. Reference [13] reports the transfer of high frequency motions to low frequencies through mechanisms of this nature. One reported experiment concerned itself with the energy transfer from an excitation signal of 194.2 Hz to a very low frequency of 3.7 Hz. The device tested was a mechanical structure constructed for this purpose. The authors designed the structure carefully trying to build up a mechanical device with its normal frequencies as multiples of each other. The transfer is given by means of the normal modes.

Reference [14] deals with normal modes in nonlinear systems, giving a detailed overview of the topic. In the field of acoustic transducers, reference [15] reports that buckling, especially at high sound levels, can lead to the production of subharmonics. This was the only cause of this type of distortion mentioned by the authors. The mode shapes of the loudspeaker diaphragms have been studied in detail by various authors [16], [17], [18], mainly because the interest of new developments including the sandwich and honeycomb core

constructions. I. Aldoshina's [19], [20] research is very specific from the point of view of nonlinear parametric oscillation and the subharmonic presence in loudspeakers. She assigns this phenomenon to diaphragms and the bending waves in the cones. Reference [21] shows a detailed overview of the results obtained when applying the Finite Element Analysis (FEA) to the titanium diaphragm and the suspension set of a 50 mm diameter tweeter. The author used the Operational Deflection Shapes (ODS) technique and the Laser Velocity Measurement (LVM) procedure as well.

3. MEASUREMENT OF SUBHARMONICS IN COMPRESSION DRIVERS

3.1. Single Subharmonic Response

In order to find the subharmonic's generation causes, four available compression drivers made by different manufacturers were tested.

Sample A is a three inch titanium dome compression driver with flat polyester suspension. Sample nominal impedance is 16 Ω . The unit was excited sinusoidally at a constant RMS value of 3 volts. The acoustic output was picked up by a microphone at the transducer's throat center in the very near field and was analyzed by the B& K dual channel spectrum analyzer type 2035. The analyzer set up was as follows:

Peak average for all sinusoidal sweep tests, this average keeps the measured amplitudes while the sweep is running. For all sweep tests, the time signal was Hanning windowed, and the overlap was set to the maximum. The displayed acoustic measurements were calibrated in dB and referred to 20 micro Pascals.

Although the sound pressure level is calibrated, several measurements gave high levels because of the proximity from microphone to the transducer. Notice this distance was as small as possible because the purpose was to pick up the signal close to the place it was produced and thus find the causes of the distorted signals.

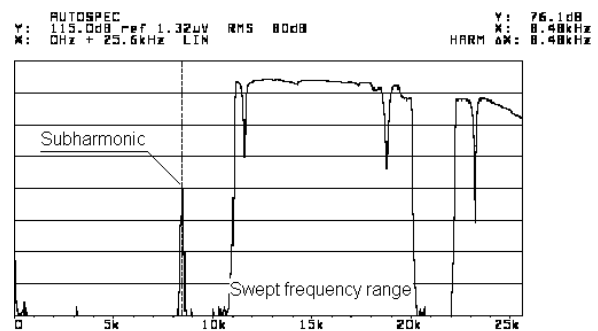


Figure 2: Acoustic band limited response of the sample A when it is swept from 11 kHz to 20 kHz. Measuring microphone installed at the output throat of the transducer. No horn used.

Figure 2 depicts the average spectrum delivered by the sample for a slow sine sweep at a constant value of three volts from 11 kHz to 20 kHz. The peaks at the right of the sweep range are due to harmonic distortion, the left peak of 76.1 dB at 8480 Hz is a subharmonic generated by the transducer under test. The sound pressure at the excitation frequency of 16960 Hz is 108.4 dB. Some cancellations in the response and in the harmonic distortion can be seen because of the position of the measurement microphone and the lack of free field conditions of the tests; but the subharmonic component is clearly enhanced.

The process of the phase lock-in of signals is accurately explained theoretically in [22], and mainly addressed to electronic devices. Reference [23] is more specific in mechanical devices, and [24] details these mechanisms both theoretically and with practical examples in several systems of different natures. In order to analyze the lock in mechanism between the electric signal and the subharmonic acoustic tone, a stable sine wave of the excitation frequency to the speaker was abruptly applied, and the response was recorded. This technique can be seen in reference [25], but the authors develop the method mathematically.

In order to avoid potential measurement errors due to the electronic equipment (heterodyning, noise addition, etc.), the measurement procedure and the instrumentation used were simplified as much as possible. The analyzer was set in the range 0 to 12.8 kHz. The instrument has a built in antialiasing low pass filter that rejects frequencies over the base band. The response of the speaker to the single forced signal was treated on the analyzer as a transient signal and recorded

and processed on time domain. P.O. Pedersen [3] found that the subharmonic does not rise immediately to its full intensity. He reported that if the excitation voltage only slightly exceeds the threshold value the speaker may take a few seconds to reach the final amplitude. When treating problems in time domain it is often convenient to use the analytic signal, instead of the signal itself, see for example the reference [26].

Following the notation of [27], if $\tilde{o}(t)$ is the Hilbert transform of the signal $o(t)$; the analytic signal corresponding to $o(t)$ is defined as:

$$\hat{o}(t) = o(t) + i \tilde{o}(t) \quad (4)$$

The magnitude or the envelope of this analytic function is:

$$|\hat{o}(t)| = \sqrt{o^2(t) + \tilde{o}^2(t)} \quad (5)$$

The (instantaneous) phase of the analytic signal is:

$$\Theta(t) = \tan^{-1}(\tilde{o}(t) / o(t)) \quad (6)$$

The rate of change of phase is the instantaneous frequency:

$$f_i(t) = 1 / (2*\pi) * d\Theta(t) / dt \quad (7)$$

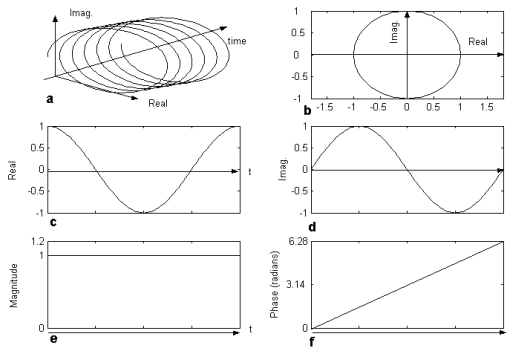


Figure 3: The analytic signal. (a) Time limited cosine function. (b) Imaginary part versus real part of the analytic signal. (c) One cycle of the real part. (d) One cycle of the imaginary part. (e) Magnitude. (f) One cycle of the phase.

Figure 3, in section a, shows the analytic signal of a finite cosine function, which in accordance with the definitions given, will have the shape of a helix revolving uniformly around the time axis. Figure 3, in

section b, depicts the imaginary part versus the real part of the analytic signal (Nyquist form). Sections c and d depict the real part $o(t)$ and the imaginary part $\tilde{o}(t)$ of a period of this analytic function $\hat{o}(t)$. Finally, sections e and f depict the magnitude which is constant along the time, and the phase which is a linear function of time increasing $2*\pi$ radians per cycle.

Figure 4 depicts the beginning of the transient time signal decomposed in the real part and the phase. Both curves exhibit a drift, which is characteristic in phase lock-in systems. The real part of the transient signal is a growing oscillation biased by the drift. H. Yabuno and his colleagues [10] obtained DC components on their experiences with parametric excited systems. The phase signal is a developing back and forth oscillation over a biased smooth ramp; the inset shows oscillations at the time of the cursor. Observe how the phase function is not the phase of a harmonic signal, which is a straight line. If we trigger the analyzer 30 milliseconds after the signal is applied to the sample under test, some parts of the lock-in process are visible. Figure 5 depicts these results over a range of 62 milliseconds, the real part is still growing, with less drift, and the phase exhibits not only back and forth oscillations but also rotations at some discrete points. The phase of the time signal during the synchronization process is performing continuous oscillations and discrete rotations until the phase ramp of a sine wave is reached.

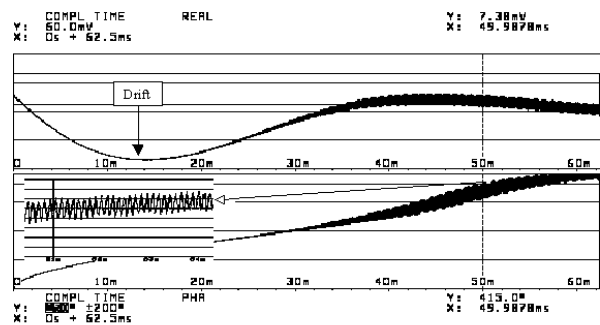


Figure 4: Sample A. Forced sine function is applied abruptly. Real part and phase of the beginning of the subharmonic response. Trigger delay of the analyzer is zero seconds. Phase inset is a detail of libration.

In classical mechanics [28] the backward and forward phase motion is called “libration” and the motion of a complete circumference, called “rotation”. Some trapeze artists on a swing are able to supply energy themselves to the swing, reaching huge oscillation

amplitudes and even a complete rotation. If the performer wants to obtain a continuous rotation, the primary motions will be various oscillations of growing amplitude before he reaches the amplitude to rotate and keep the rotation, which is the final steady state. In [24] librations are named “forced kicks”, and the authors explain that this is a very common way to synchronize systems. These discrete rotations of the phase signal, as shown in Figure 5, seem to be discrete unstable switches, but the signal is not mature enough to keep the final phase state that is linear phase increment and returns to the lock-in process by periodic kicks. The authors of [24] called these transitory complete rotations “slips”, and they explained it as synchronism loss or tuning loss. The slips of the Figure 5 seem to be an adjustment from the phase to the amplitude. When the subharmonic signal is growing the best adjustment for the next time instant is with an amplitude, very similar to the previous one, and a phase rotation (one or several complete turns). This is the best synchronization because the amplitude of the oscillation is more stable than the phase. If the conditions of permanent rotation have not been achieved then the slave oscillator (the subharmonic mode) will stay in forced kick mode until it does achieve it.

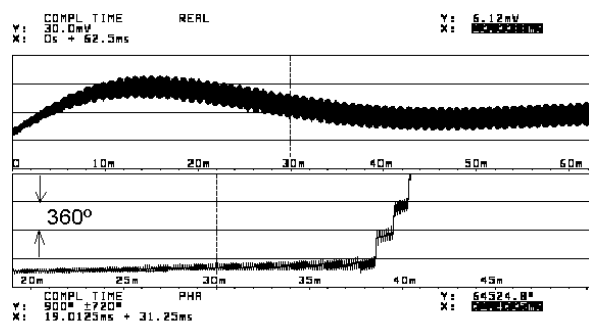


Figure 5: Sample A. Forced sine function is applied abruptly. Real part and phase of the subharmonic response. Trigger delay is 30 milliseconds. Phase shows “slips” or discrete rotations.

In reference [29] one of the authors, A. Neishtadt, explains that the change from libration regime to rotation regime or vice versa is a resonance capture. This means that the discrete rotations shown in figures

are unstable transient resonance captures, previous to the steady state of a stable subharmonic.

3.2. Measurements in a Larger Diameter Unit

Sample B is a four inch dome compression driver with bellows suspension. The suspension and the dome are made of titanium. The nominal impedance is 8 Ω. This unit presented two subharmonics for an excitation level of 3 volts.

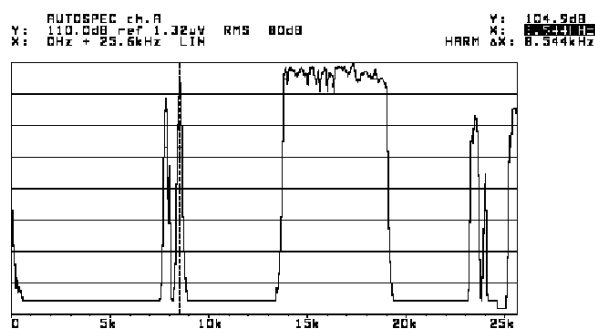


Figure 6: Sample B. Acoustic band limited frequency response when sample is swept from 13 kHz to 20 kHz. Microphone is on the near field of the suspension.

In order to pick up the signal as close to its origin, it was decided to implement the measurements very close to the moving assembly. The back cover was taken off, and because the highest subharmonic levels were found close to the suspension, the measuring microphone was installed in front of the suspension at a distance of 2.5 mm. Following the same procedures completed for sample A, a slow sine sweep of 2 volts RMS was applied to the unit. The region swept was 13 kHz to 19 kHz. The result is illustrated in Figure 6 where two differentiated subharmonic peaks are shown. The subharmonic frequencies are 7800 Hz and 8540 Hz respectively. Because of the proximity of the microphone to the region of subharmonic generation, the relative level of these components in respect to the sweep is much higher than those found in sample A.

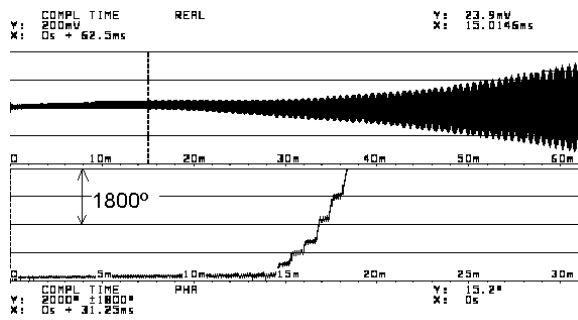


Figure 7: Sample B. Forced sine function of 17106 Hz is applied abruptly. Real part and detail of phase of the beginning of the subharmonic response. Measurement microphone was installed very close to the suspension.

The steady sine signals, whose frequencies are double that of the subharmonic components, were applied following the procedure used in sample A, and the “birth” of the subharmonics was recorded. Figure 7 depicts the first part of the time signal of the formation process for the upper frequency (17106 Hz). Observe how the amplitude drift for this sample is much lower than that of sample A, but now the signal is picked up on the very near field of a single part of the transducer. The small amplitude oscillations of this figure and the following are caused by the digital analyzer shortage of sampling; but the use of this analyzer has the advantage of simplified measurement instrumentation, which is very convenient for the registered transients.

The transient rotations of the time phase between 15 and 20 millisecond are integer multiples of a complete rotation. The rest of the figure is similar to Figure 5. Figure 8 depicts the formation of the subharmonic component, close to the final stable state, much later. Observe how the amplitude is still growing but the phase has reached the final stable state with a pure rotation (linear increasing phase). Notice in the figure that the phase is rotating 3097.6 degrees each millisecond. This phase corresponds to a frequency of 8543 Hz. It is necessary to take into account the different vertical scales of the phase functions of Figures 4, 5, 7 and 8, in order to understand the “birth” of the subharmonic signals and compare the different phases of the process.

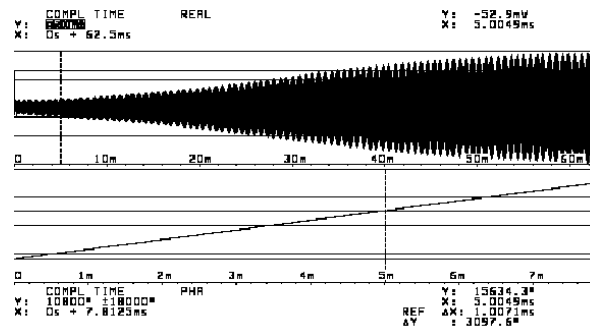


Figure 8: Sample B. Forced sine function of 17106 Hz is applied abruptly. Upper: Real part of the time response of the final part of the subharmonic formation. Lower: Phase of the time response of the final part of the subharmonic formation.

Because both subharmonics grow at different speeds, additional measurements have been taken to show their differences. Figure 9 depicts the real part of both subharmonics, being triggered by the analyzer 60 milliseconds after the application of the excitation signals to the sample. The transient shown at the bottom has a settling time much shorter than the one represented on top of the figure. Notice this fastest subharmonic seems to have a bounded output, while the slower subharmonic is growing much more slowly (compare the scales in legend on the left side of the graph), and is far from this bounded output. Notice too, the small drift which the slower subharmonic has, as shown on the left portion of the magnitude function.

Amplitude oscillations of Mathieu’s type [3] for real systems with damping are controlled by the balance between the external energy fed to the system and the losses of the system. It seems that the process follows a limit cycle scheme with the amplitude increasing until it reaches the cycle, which is stable. The theory of limiting cycles is available in various books; see for example [30] and [31]. An acoustic limit cycle is mentioned in [32]. W.J. Cunningham [5], in 1951, explained in his paper that there is a “limited action” of the growing subharmonics.

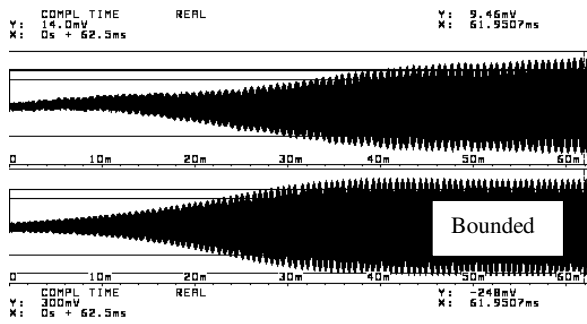


Figure 9: Sample B. Comparison of real part time responses of the slow (upper) and fast (lower) subharmonic when the analyzer is trigger delayed by sixty milliseconds after the delivery of the electric sine signal.

It may be convenient to measure the electromotive force induced on the speaker’s coil while the subharmonic is fully generated in steady state; this gives information of the coil’s axial velocity during the subharmonic radiation. The unit was excited with 2 volts. Figure 10 depicts frequency spectra of the voltages on the coil while the sample is radiating each subharmonic. Observe how the fastest subharmonic has a level 18.8 dB (8.71 times) lower than the slow one. This higher axial coil motion of the slow subharmonic may indicate that this subharmonic has higher probability to involve the transducer’s axial mode (shown below in the text) than the fast subharmonic.

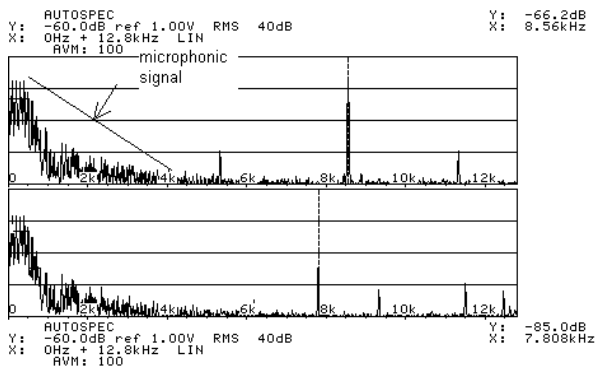


Figure 10: Sample B. Spectral analysis of the coil’s induced voltage, in steady state, while the driver is radiating the slow (upper) and fast (lower) subharmonic.

3.3. Modal Analysis of the Whole Moving Assembly

The authors P.O. Pedersen [3], H.F. Olson [4], W. J. Cunningham [5] and J.K. Hubbard [6], believed that an important cause of the subharmonic generation is related to diaphragm resonances. D. Bie [21] included in his study the dynamics of the compression driver’s suspension. The performance of the transducer must include the coil and the former as well because they have a definite contribution to the moving assembly dynamics.

The moving assembly has been modeled by means of finite element software. The models use triangle elements of the Kirchoff type, except for the dome modeling that was a quadrilateral element. The element used carries five degrees of freedom per node (three translations and two rotations). This element is suitable for shells that carry in-plane and out-of-plane loads. Those modes related with the dome deformation are better found with a square element for symmetry causes. The software provides modal analysis of the moving assemblies.

There are some mode shapes of the moving assembly that can carry the subharmonic tones (the response related with ω_0), and there are modes able to act as excitation modes (the cause related with the pumping p). A brief basic description of the significant modes involved in this process is the following:

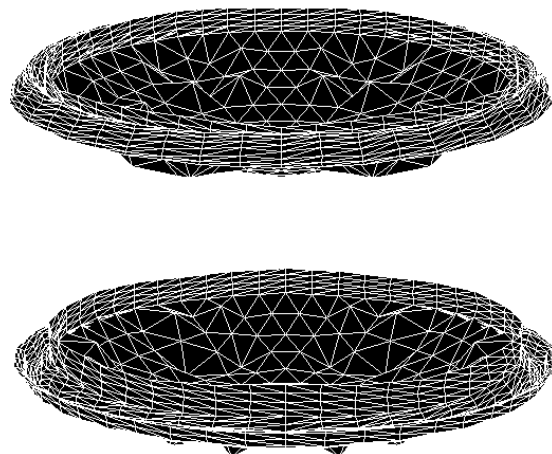


Figure 11: Main suspension mode. The suspension flaps up and down, the rest of the moving assembly is nodal or almost nodal.

a) Main Suspension's Flapping Mode. The suspension flaps axially up and down with all points moving in phase. The suspension outer circumference is obviously nodal and the inner circumference is nodal or nearly nodal too. Below suspensions are more excitable in this mode than the flat ones. The median circumference of the suspension is an antinode. Figure 11 depicts this mode. This mode is relevant because the axial force on the coil it is concentric with inner suspension's circumference.

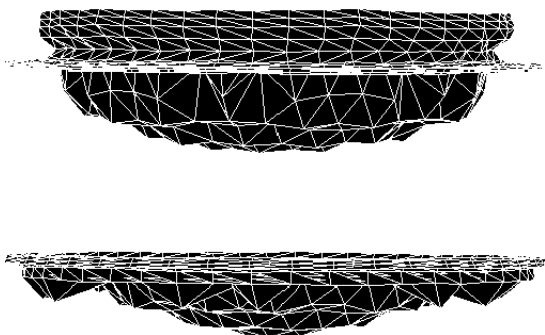


Figure 12: Transducer axial mode. The coil and the dome move axially towards each other. The former shortens and stretches axially.

b) Main Axial Mode of the whole moving assembly. This mode consists in the axial stretching and compressing of the whole former (which is weak in respect to the neighboring materials), because of the inertia forces of the dome and the coil moving along the transducer axis. These masses move like a two degrees of freedom system in its second mode, axially pushing and pulling the former. Depending on the former's thickness the mode may include a certain amount of "breathing" of the main inertias in motion (the dome and the coil). This mode is relevant because the force is exerted on the coil in the same direction that the coil's has in the mode shape. The mode is illustrated in Figure 12.

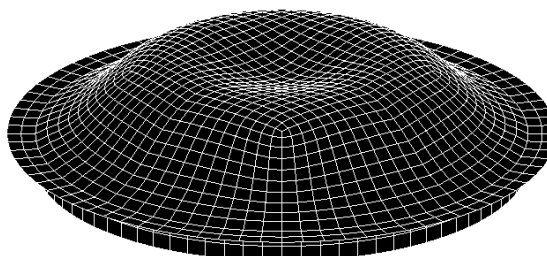
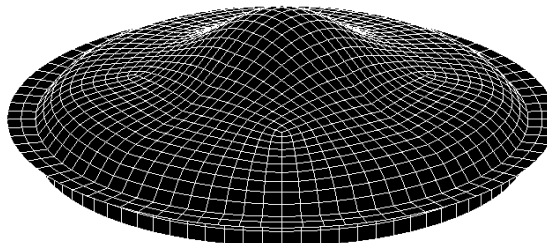


Figure 13: Circumferential axisymmetric dome mode with one nodal circumference.

c) Symmetric Dome Modes. The dome exhibits modeshapes that are symmetrical as in a disk, following the terminology of Mac Lachlan [33]. The author classified the mode shapes in a diaphragm as symmetrical (as in a disk) and radial (as in a bell). The symmetric modes are circumferential modes; which nodes and antinodes are circumferences. These symmetric modes yields substantial axial forces and or moments at nodal circles, and these modes are able to excite other modes which have the same nodal circles too. In particular, the coil and the inner suspension circumference are nodal for these mode shapes. Two of these modes are depicted in the Figures 13 and 14. These modes are easily excited in a compression driver because the coincidence with the shape of the phase-plug slots, and because the strong air load at which the dome is submitted. Domes of large diameter have higher probability to be excited by these modes because the shallow arch behavior [40] and [41].

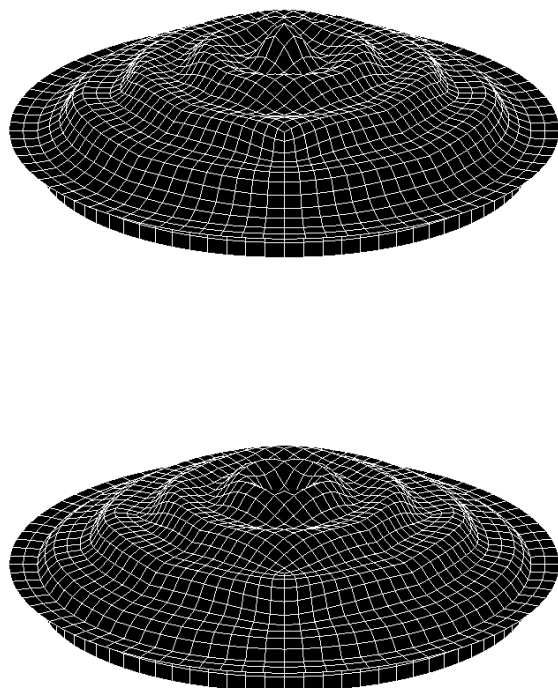


Figure 14: Circumferential axisymmetric dome mode with various nodal circumferences.

d) Coil's Conical Mode. The set composed by the coil and the former becomes conic, with the apex switching up and down. The mode shape is illustrated in Figure 15. This mode is not very much influenced by the stiffness of the nearby suspensions and consequently is not very sensitive to or dependent on, the suspension shape. The mode shape is axisymmetric.

e) Coil's "Breathing" Mode. It is a radial mode shape based on the so-called "in-plane" forces acting on the coil. These in-plane forces are forces of each coil constituent element stretching and shortening the coil like a belt. A simple illustration of this motion could be the deformation obtained when heating and cooling the body periodically. The coil opens and closes as if it was "breathing". These concepts are explained in the books [34] and [35]. The coil's breathing mode, which is an extensional mode, is illustrated in Figure 16. The mode shape is axisymmetric but normal to the coil's force.

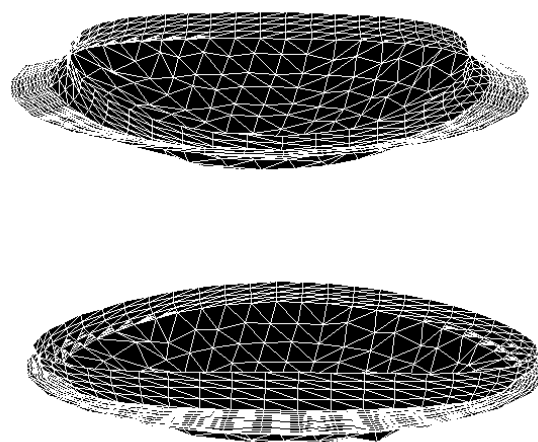


Figure 15: Coil and former conic mode. The coil and former closes and opens depicting a conic shape with the apex swinging up and down. The suspension moves synchronously following the former.

Any of these modes can be a subharmonic or motional mode (the mode which radiates sound).

The acoustic efficiency of the subharmonic will be proportional to the modal antinode area and the position of this antinode area in respect to the transducer axis. Any of the significant modes mentioned above can be an excitation mode if the action is autoparametric. This excitation mode will supply the periodic change (linear or rotary) of stiffness or length to the motional mode. If a nodal contour line of a radiator mode receives forces or moments in phase that modify the average translation or rotation stiffness of this line, fulfilling the Mathieu's conditions (for example twice per cycle), then a parametric action is established. If the external force or moment modifies the average length of angle of this contour line, this will cause a parametric action too. When the external force is exerted by the coil because of the electromagnetic force, then there is a parametric excitation. If the coil's force excites any mode of those mentioned above and this mode feeds force or moment, by geometrical coupling, to the nodal contour line of one of the radiator modes, then the oscillation will be autoparametric. When a system or device has autoparametric oscillations it's said that has internal resonance.

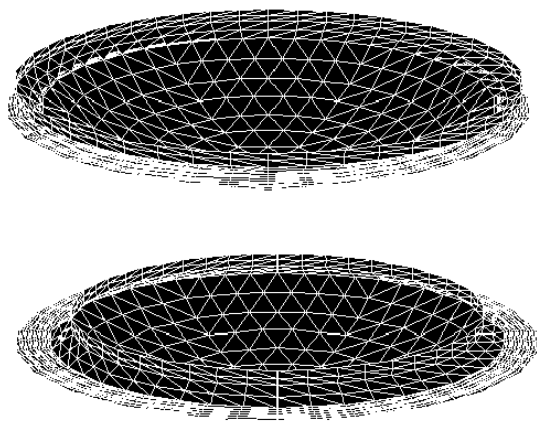


Figure 16: Coil in extensional mode or breath mode. The coil opens and closes on a plane which is perpendicular to the transducer axis. The former follows the coil changing periodically its slope.

The coupling of modes, as mentioned before, is done through nonlinear mechanisms, some of which are not fully understood, [12] and [13]. Usually in compression drivers these couplings are not linked too fast and need some synchronization time, as depicted in Figures 4, 5, 7 and 8.

It is obvious that those modes which the main motion is on the transducer axis are more excitable than those which do not, because the coil's action tends to excite these symmetric modes. The conic mode and the coil's breath mode, despite the main motions are not in the axial direction, they can be excited because in practice there is a residual axial motion of these modes (the coil's axial motion is small but not null) and because they are much less damped by the electric forces than the modes with large axial motion of the coil.

4. OTHER RELATED DISTORTIONS

4.1. Spontaneous Sidebanding Response

Sample C is a three inch titanium dome compression driver with flat polyester suspension. This transducer is similar to sample A, but it was made by a different manufacturer. Nominal transducer impedance is 8 Ω. This sample did not have the tendency to generate subharmonics with the electric voltages we submitted to

sample A. Nevertheless for excitation voltages equal or higher to 4 volts, the sample exhibited spontaneous sidebanding. Measurements of sidebanding response in compression drivers were reported in [6], although the causes of the phenomenon were not explained. A sidebanding response is called spontaneous, because these sidebands are not produced by intermodulation of two tones applied to a nonlinear device. On the contrary the excitation is only a single frequency. The term “spontaneous sidebanding” has been taken from other fields of engineering [44]

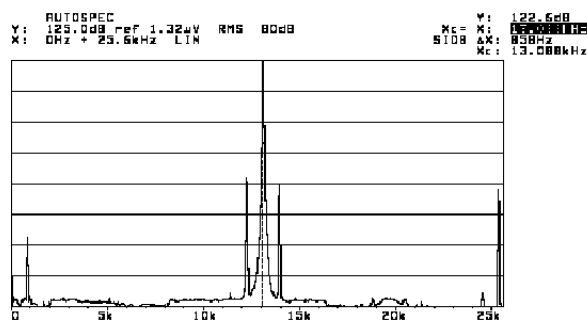


Figure 17: Spontaneous sidebanding performance when sample C is excited with a sine wave of 13088 Hz.

In Figure 17 there is the response of sample C to a stable sinusoidal signal of 13088 Hz in which we can see the frequency of the applied signal and two sidebands 858 Hz apart. Observe as well, on the right side of the figure a sideband of the second harmonic of the applied signal (the second harmonic is beyond the analyzer frequency range). The transducer responds as if it had been excited by the high and low frequencies simultaneously.

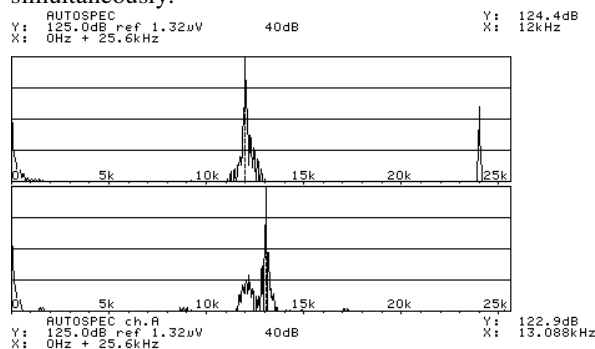


Figure 18: Fourier transform of the acoustic response of the sample C for a sine wave abruptly applied. Upper figure is the response for a 12 KHz excitation, and the lower is for 13088 KHz.

When sample C is excited with sine waves applied abruptly to the transducer, the compression driver responded as depicted in Figure 18. One of the testing frequencies was 12 kHz, because this frequency does not give sidebandings when it is steadily applied to the unit. The frequency of 13088 Hz which responded with sidebandings was applied too. Care was taken to record only the transient response and not the steady state signal. Observe at the bottom of the figure the spectral response of the speaker to the sidebanding frequency, which gives a noisy left side lateral hump in the region of one of the sidebands of the steady state. The signal of 12 kHz only responds with a single, but noisy, base spectral line.

The one dimensional bilinear oscillator is reported theoretically and practically by J.T. Anderson and others [36] who worked exciting a single beam. E.L.B.Van der Vorst [37] reported the response of a nonlinear system giving a subharmonic response, a quasi-periodic response or a chaotic response. It refers to a mechanical system made up of a cantilever beam to which a nonlinear element has been placed in the center. The nonlinear element is what the author has named a “one sided spring”. R.S. Chancellor and others [38] published measurements of an excited beam with a mechanical stop on top of the flexible beam.

P.V. Bayly [39] defines a weakly bilinear oscillator as an oscillator that behaves linearly in each of two regions, but which has at least one parameter which differs by a small amount in one region. The parameter is discontinuous at the boundary between regions. Figure 19 depicts a bilinear oscillator with its motion constraint to a line. The oscillator responds to the analytic expressions of (8):

$$d^2 x / dt^2 + \omega_0^2 x = A \sin(\Omega * t), \quad x \geq 0 \quad (8a)$$

$$d^2 x / dt^2 + (\omega_0^2 + \delta) * x = A \sin(\Omega * t), \quad x < 0 \quad (8b)$$

Being:

x: the displacement

ω_0 : the natural circular frequency for the zero and positive displacement

Ω : the forced circular frequency

δ : the square of the detuning circular frequency (being small compared with ω_0^2)

A: the amplitude of the external forcing function.

For an oscillator as the one shown in Figure 19 governed by the equation (8), it is reasonable to have sidebanding responses as the system’s natural frequency is switching when the mass crosses zero displacement. Observe how in this bilinear oscillator of the equation (8), just as in Mathieu’s formula, it separates to the form of a linear single degree of freedom system on the stiffness terms. In a compression driver the light diaphragm is submitted to a heavy load and it can bounce over the air in the phase plug gap as a bilinear oscillator.

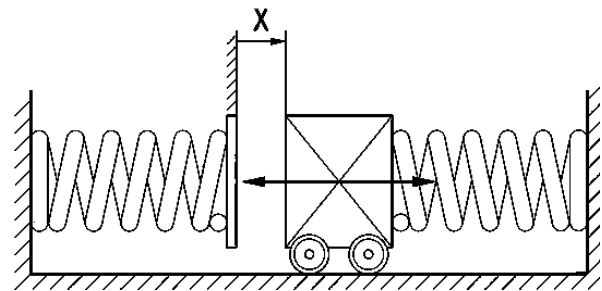


Figure 19: Bilinear oscillator constrained to move along a single line.

In reference [40], Soliman and Gonçalves demonstrates the steady state instabilities that may occur in shallow spherical shells. These instabilities include jumps to resonance, subharmonic periodic doubling bifurcations, cascades to chaos, etc. The work of Amabili and Paidoussis [41] summarizes 356 references that deal about shell’s dynamics, including the nonlinear phenomena.

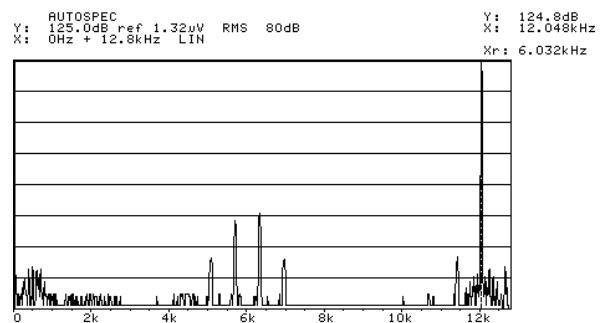


Figure 20: Response of sample D with spontaneous sidebanding in the half frequency range of the excitation signal. Transducer is excited by a sine wave of 12048 Hz. Sample D has an annular V-shaped diaphragm.

4.2. A Spectral Fractal Structure

Sample D is a coaxial compression driver. The diaphragm is a polyester V shaped ring. The coil has a diameter of 90 mm, and the moving assembly is suspended by a polyester bellows suspension. The transducer nominal impedance is 8 Ω . Only the mid-range transducer of the coaxial set was tested. Measurements were taken with the microphone on the near field of the transducer's throat.

The sample showed a spontaneous sidebanding response in several spectral regions. A remarkable characteristic of this sample is the presence of spontaneous sidebanding in the half frequency region of the excitation. Figure 20 depicts the response of sample D to a sine tone of 12048 Hz with an amplitude of 3.7 volts and with a rather defined spectral structure on its base. A significant response of the compression driver to the single excitation frequency is the spontaneous sidebanding developed at half the excitation frequency. This response is a sort of combination of the subharmonic and bilinearity character of the unit for this specific excitation frequency.

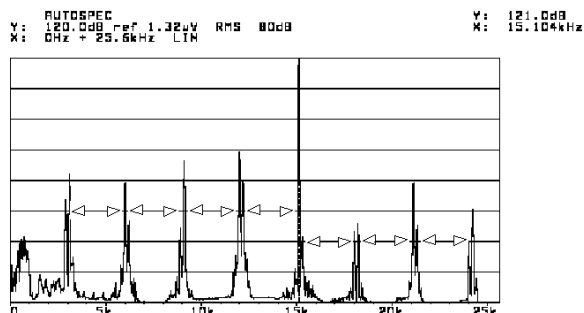


Figure 21: Spectral fractal structure of sample D. Excitation is a sine wave of 15104 Hz. The frequency analysis covers the full transducer range.

One of the testing sine signals with a frequency of 15104 Hz gave a spontaneous wide band sidebanding. Figure 21 depicts the spectrum response to this single frequency. As most individual peaks of this figure exhibit several spectral lines, a zoom-in of some peaks will detail the internal spectral structure. The zone of 12 kHz is one of the broadest, and was analyzed with a higher resolution. Figure 22 depicts the spectral structure of this region. A new closer sidebanding is visible with some spurious peaks. As the central peaks

depicted broad bandwidth at its bases, a new finer analysis was performed. The result is in Figure 23, where again the sidebanding is evident. The process is not stable enough, as the moving assembly is light weight, to capture the next spectral structure, and a finer spectral definition needs longer time record in the analyzer.

This repetitive structure is called a fractal structure or self similarity structure. M. R. Schroeder [42] and W. Lauterborn and others [43] explained this concept and they included acoustic examples. While the compression driver is radiating an acoustical fractal structure, some parts or the whole moving assembly are probably bouncing in such a way that self similar motions are being described. These motions are nearly impossible to enhance in the time domain, but are clearly visible in the frequency domain depicted on the fractal spectra.

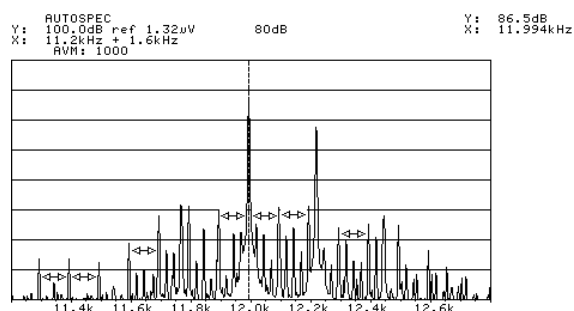


Figure 22: Spectral fractal structure of sample D (second generation). Excitation is a sine wave of 15104 Hz. The zoomed analysis is centered at 12 kHz.

The bilinearity is a severe nonlinear condition. One characteristic of the high nonlinear systems is that they are very dependent on the initial conditions (initial rebound). This sample D showed a low level test repetition. This is reasonable because the repetition of the test does not guarantee to have exactly the same initial conditions. Besides, and for the same reasons, we obviously cannot be sure of finding the same results when testing the unit with its axis in a horizontal or vertical position. The transducer's temperature also changes the contact stiffness. When the laboratory's temperature decreases just 6 degrees centigrade, it is necessary to shift the generator frequency from 15104 Hz to 15257 Hz to obtain the fractal structure tuning.

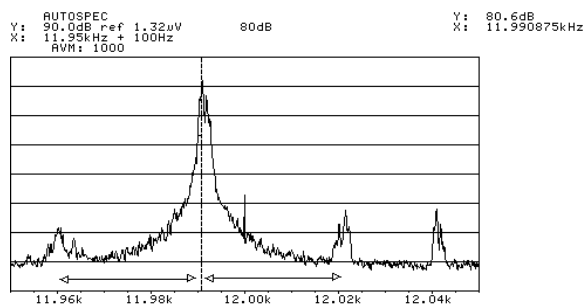


Figure 23: Spectral fractal structure of sample D (third generation). Excitation is a sine wave of 15104 Hz. The frequency analysis is a zoom-in of the spectral peak in the 12 kHz range shown in Figure 22

5. CONCLUSIONS

Two of the four tested samples showed subharmonic distortion. The analysis of the time domain response illustrates the synchronization process between the electric applied signal and the subharmonic acoustic output. Modal analysis of the moving assembly demonstrates that there are various mode shapes that can cause the parametric or autoparametric (internal resonance) excitation and response of the compression drivers. When we try to find the causes of this distortion, apart from the diaphragm dynamics, the coil and former's dynamics and the suspension's dynamics must be taken into account as well.

Two of the four samples do not exhibit subharmonic responses but showed spontaneous sidebanding. This sidebanding response is attributed in literature to bilinearity. The bilinearity can be caused by the compression process itself, because a light and weak diaphragm must compress air in a shallow volume and in a spectral range where the diaphragm has several natural frequencies.

One of these two samples responded with a self similarity motion or fractal pressure response. These spectral structures can be found in theoretical dynamics, formal laboratory experiences and in machinery dynamics, but very seldom [6] in the field of acoustic transducers.

It is difficult to classify these subharmonic distortions in a systematic manner, which is not the case with the harmonic and intermodulation distortions we are used to working with. The assessment of the influence of these

distortions in the final acoustic performance of the transducer seems to be at a very early stage.

6. ACKNOWLEDGEMENT

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7. REFERENCES

- [1] M. Faraday, On the forms and States Assumed by Fluids in Contact with Vibrating Elastic Surfaces, *Philosophical Transactions of the Royal Society*, London, 1831, Vol. 42, pp 319-340.
- [2] G. Floquet, Sur les equations différentielles linéaires à coefficients périodiques, *Ann. Sci. Éco. Norm. Sup*, Vol. 12, pp 47-89, 1883.
- [3] P. O. Pedersen, Subharmonics in Forced Oscillations in Dissipative Systems. Part I, *Journal of the Acoustical Society of America*, Vol. 6, April 1935; and Part II, Vol. 7, July 1935.
- [4] H. F. Olson, *Acoustical Engineering*, Professional Audio Journals, Inc., Philadelphia, Pennsylvania 1991. First publication by D.Van Nostrand Co. Inc. 1957.
- [5] W.J. Cunningham, The Growth of Subharmonic Oscillations, *Journal of the Acoustical Society of America*, Vol. 23, Number 4, pp 418 – 422, July 1951.
- [6] J. K. Hubbard, Subharmonic and Nonharmonic Distortions Generated by High Frequency Compression Drivers, *AES 6th International Conference: Sound reinforcement*, May 1988.
- [7] D.W.Jordan and P.Smith, *Nonlinear ordinary differential equations*, Oxford University Press, 1977.
- [8] A.H. Nayfeh and D. T. Mook, *Nonlinear Oscillations*, John Wiley and Sons, 1979.
- [9] R. Dufour and A. Berlioz, Parametric Instability of a Beam Due to Axial Excitations and to Boundary Conditions, *Journal of Vibrations and Acoustics*, Vol. 120, April 1998.
- [10] H.Yabuno, Y Endo, N. Aoshima, Stabilization of 1/3 Order Subharmonic Resonance Using an

- Autoparametric Vibration Absorber, Journal of Vibration and Acoustics, Vol. 121, July 1999.
- [11] P.H.Nguyen and J.H.Ginsberg, Vibration Control Using Parametric Excitation, Journal of Vibration and Acoustics, Vol. 123, July 2001.
- [12] S.A. Nayfeh and A. H. Nayfeh, Energy Transfer From - High to Low – Frequency Modes in a Flexible Structure via Modulation. Journal of Vibration and Acoustics, Vol. 116, April 1994.
- [13] A. H. Nayfeh and D.T. Mook, Energy Transfer from High – Frequency to Low – Frequency Modes in Structures, Journal of Vibration and Acoustics, Vol. 117, June 1995.
- [14] A.Vakakis and others, Normal Modes and Localization in Nonlinear Systems, Wiley, New York 1996.
- [15] D.A. Barlow, G.D.Galletly and J. Mistry, The Resonances of Loudspeaker Diaphragms, Journal of the Audio Engineering Society, October 1981.
- [16] N. Sakamoto and others, Loudspeaker with Honeycomb Disk Diaphragm, Journal of the Audio Engineering Society, October 1981.
- [17] K. Suzuki, and I. Nomoto, Computerized Analysis and Observation of the Vibration Modes of a Loudspeaker Cone, Journal of the Audio Engineering Society, March 1982.
- [18] T.Yamamoto and others, High Fidelity Loudspeakers with Boronized Titanium Diaphragms, Journal of the Audio Engineering Society, December 1980.
- [19] I. Aldoshina and others, Theoretical and Experimental Analysis of Nonlinear Parametric Vibrations of Electro-dynamical Loudspeaker Diaphragm, 104th Convention of AES, May 1998.
- [20] I. Aldoshina and others, An Advanced Model of Nonlinear Parametric Vibrations of the Electrodynamical Loudspeaker Diaphragm. 106th Convention of AES, May 1999.
- [21] D. Bie, Vibration Resonances of a Titanium Loudspeaker Diaphragm. 104th AES Convention, Amsterdam May 1998.
- [22] F. M. Gardner, Phaselock Techniques, J.Wiley and Sons, April 1979.
- [23] A. B. Pippard, The Physics of Vibration, Cambridge University Press, 1979.
- [24] A. Pikovsky, M. Rosenblum and J. Kurts, Synchronization, a universal concept in nonlinear sciences, Cambridge University Press, 2001.
- [25] H.L.Neal and A. H. Nayfeh, Response of a Parametrically Excited System to a Nonstationary Excitation, Journal of Vibration and Control, Vol. 117 , n°1, 1995.
- [26] A. Duncan, The Analytic Impulse, Journal of the Audio Engineering Society, Vol. 36, N° 5, May 1988.
- [27] N. Thrane, The Hilbert Transform, Technical Review , Edited by Brüel & Kjaer, N° 3, 1984.
- [28] H. Goldstein, Classical Mechanics, Addison Wesley Publishing Co., 1980.
- [29] D. K. Campbell (editor), Chaos, Soviet – American Perspectives on Nonlinear Science, Edited by the American Institute of Physics, New York 1990.
- [30] K.Ogata, Modern Control Engineering, Prentice Hall Inc., 1970.
- [31] H. T. Davis, Introduction to nonlinear differential and integral equations, Dover Publications 1962.
- [32] R.R. Erickson and B.T. Zinn, Modeling of finite amplitude acoustic waves in closed cavities using the Galerkin method, Journal of The Acoustical Society of America, Vol. 113, n° 4 April 2003.
- [33] N. W. McLachlan, Loudspeakers: Theory, Performance, Testing and Design, 1934, Oxford University Press.
- [34] F. Fahy, Sound and Structural Vibration, Radiation Transmission and Response, Academic Press, 1985.

[35] M.C.Junger and D. Feit, Sound, Structures, and Their Interaction, Acoustical Society of America, 1993. First edition in 1972.

[36] T. J. Anderson, B. Balachandran and A. H. Nayfeh, Nonlinear Resonances in a Flexible Cantilever Beam, Journal of Vibration and Acoustics, Vol. 116, n°4, 1994.

[37] E.L.B.Van der Vorst and others, Vibration Control of Periodically Excited Nonlinear Dynamic Multi-dof Systems, Journal of Vibration and Acoustics, Vol. 117, n°1, 1995.

[38] R. S. Chancellor, R.M. Alexander, S. T. Noah, Detecting Parameter Changes Using Experimental Nonlinear Dynamics and Chaos, Journal of Vibration and Acoustics, Vol. 118, n°3, 1996.

[39] P. V. Bayly, On the Spectral Signature of Weakly Bilinear Oscillations, Journal of Vibration and Acoustics, Vol. 118, n°3, 1996.

[40] M. S. Soliman and P.B. Gonçalves, Chaotic Behaviour Resulting in Transient and Steady State Instabilities of Pressure – Loaded Shallow Spherical Shells, Journal of Sound and Vibration, vol. 259 (3), 2003.

[41] M. Amabili and M. P. Paidoussis, Review of Studies on Geometrically Nonlinear Vibrations and Dynamics of Circular Cylindrical Shells and Panels, with and without Fluid-Structure Interaction, Applied Mechanics Review, Vol. 56 (4), July 2003.

[42] M. R. Schroeder, Self-Similarity and Fractals in Sciences and Art., Journal of the Audio Engineering Society, Vol. 37, October 1989.

[43] W. Lauterborn and U. Parlitz, Methods of Chaos Physics and Their Applications to Acoustics, Journal of Acoustical Society of America, Vol. 84, n° 6, December 1988.

[44] F.F. Eirich, Spontaneous Sidebanding in High Speed Rotordynamics, Journal of Vibration and Acoustics, Vol. 114, October 1992.