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Modal Analysis and Nonlinear Normal Modes (NNM) on Moving Assemblies of Loudspeakers

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ABSTRACT

The most important modes for a direct acoustic radiator are the axial modes, which are axisymmetric circular modes of a high temporal and spatial coherence [38]. Numeric modal analysis and measurement of the free and forced accelerations and displacement responses of the moving assemblies are performed to establish the main modes involved in the acoustic response. The axial modes had been identified by measurements (within the intrinsic degree of uncertainty). The experiences show evidence of clearly nonlinear normal modes (NNM) [18] and [19], justifying the high complexity of mode finding in loudspeaker cones. Based on the axial modes, a three degrees of freedom model is proposed, where only one of the masses is externally forced. The modal analysis of a double cone speaker has been treated in short form.

1. PURPOSE AND INTRODUCTION

The intention of this paper is to find some relevant peaks of the acoustic frequency response and harmonic distortion of woofers by means of motion measurements and numerical modal analysis of the complete moving assembly of these transducers. If the loudspeaker designer determines the main modes of the moving assembly, both its frequencies (eigenvalues) and their mode-shapes (eigenvectors), it will be easier to fulfill the design goals of the project.

The problem of waves in cones has been treated thoroughly by Y. P. Guo [1] who explained why the waves have larger amplitudes near the apex than in the rest of the cone, based in the principle of energy conservation. The same author in [2] deals with the fluid load influence on the conical shells, in the paper Y.P. Guo explains that compressional and flexural waves are significantly affected by fluid loading. For compressional waves, fluid-loading short circuits the cutoff phenomenon. In the loudspeakers field an intense work has been done with respect to the dynamic response of the cones and the acoustic radiation of the diaphragms, see for example the references [3] and [4]. Frankort [3] completed an entire experimental job and he reported *"there is a great variation in velocity amplitudes at the inner edge of the cone"*, but essentially, the author did not pay enough attention to the voice coil. Kaizer [4] did the job numerically and he said *"as the frequency is increased the transverse velocity of the cone surface becomes non – uniform since the amplitude of the vibration increases towards large radii"*. Despite the fine approach work he did, the voice coil and former influence were not treated profoundly enough.

However the importance of the coil on the moving assembly dynamics was earlier reported by N. W. McLachlan [5]. The author mentioned the coil dynamics as well as the cone; he dealt with what he calls "optimum mass of moving coil", "influence of coil mass on acoustic output and frequency of vibration", etc. McLachlan also referred to the radial modes and symmetrical modes in the cone. These symmetric modes are relevant in the transducer performance. In F. V. Hunt's book [6] the author refers to R. L. Wegel of Western Electric (who held the U.S. patent nº 1926888 in 1924) and says that *Wegel while observing the queer motion of a too long driving link in the motor mechanism of a primitive loudspeaker claimed that "to describe that motion in terms of only six degrees of freedom would be a gross oversimplification"*. Moreover this oversimplification assumes the knowledge of the rocking eigenfrequencies of the speakers, which is often not the case. By means of the finite element method, Shindo and his colleagues [7] were able to study the influence of the voice coil on the acoustic response.

The present work has been done with fifteen inch woofers for two main reasons. First the size and weight of the moving assembly allow an experimental approach. On the other hand the elasticity of the interface cone and former, together with the cone and coil masses give moderately low natural frequencies, which can still be measured with standard instruments available in laboratories. The experimental work has been done with straight cones and in the small amplitude signal range. The problem of motion in the large is beyond the scope of this paper.

2. BASIC MODE SHAPES ON MOVING ASSEMBLIES

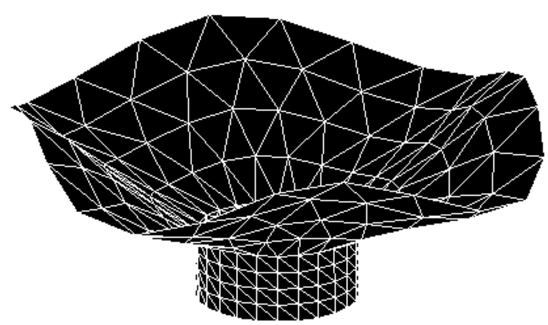
Whereas in a uniform thin flat plate, small-amplitude bending waves and in-plane waves are uncoupled and can propagate independently; in a curved plate the different type of waves are coupled.

There are two important modes in a cone, see for example, Barlow [8] and Krüger [9]:

a) Radial modes, McLachlan called them bell modes [5]. Radial modes consist in the sectors of the cone moving in opposite directions. The radii between the sectors are antinodes. Acoustic cancellation takes place by lateral movement of air across the face of the cone

b) Circular modes, McLachlan called them symmetrical modes [5]. These modes consist in the motion of the cone with ring shape with antinodes between the concentric rings. These modes are strongly coupled to the voice coil by the axial motion of it, and are the cause of relevant peaks on the acoustic response.

The most common terminology [10] is the classification of the modes according to the number of nodal circles and nodal radii (on some texts referred to as meridians) the cone has (NC, NR). So that any radial mode has NC equal to zero and any circular mode has NR equal to zero as well. Two of these modes are depicted in Figure 1. The two lowest element rows of the figure belong to the coil while the two element rows on top of it are the former.



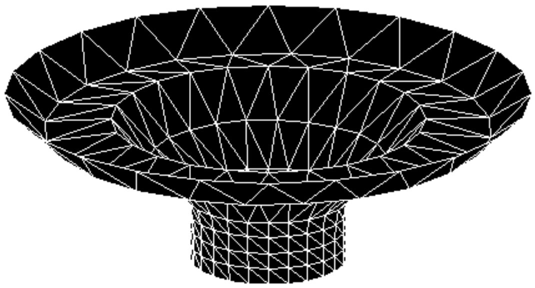


Figure 1: Radial mode (top) and circular mode (bottom) on a moving assembly of a direct radiator.

In a cone near the apex, the in-plane forces (membrane forces) dominate the response, while near the outer diameter the bending flexibility is significant. Despite the importance of the inertia of the voice coil and the former compliance, relevant deformations are on the cone and in the outer suspension as well.

Besides these circular and radial modes of the cone, the addition of the voice coil – former set to the cone, and the addition of the two suspensions, configures a more complex situation with other relevant modes than are dealt with in this paper This offers a better understanding of the moving assembly dynamics.

2. 1. Main Axial Modes

If we apply sufficient DC voltage to the voice coil in such a way that the suspension stiffness of the speaker reaches the practical infinite value, at the end of the negative stroke; the cone, former and coil set trend to stretch axially as depicted in the Figure 2. In the figure, we can see that there are two regions at which the cone has the maximum deformation. A high strain is exerted circumferentially on the cone body near the rim edge. The figure illustrates that the neck deforms as well. Whereas the outer cone ring deforms basically axially and bending, the former strains bending, and the inner cone ring strains both axially and bending. When inertia forces, the damping forces and the elastic forces, together with the external forces, act simultaneously in a linear moving assembly, while the speaker is excited axially; the moving assembly will exhibit, essentially, two regions acting as a spring and three acting as a mass. The deformed regions of the figure are the springs of the body, one is near the rim and other is in the neck region, generally the stiffness of the neck is different than the one of the rim zone. The masses are the bodies which are separated by the springs. Axial motions are the most temporal and space coherent motions [38] of the moving assembly giving maximal acoustic radiation.

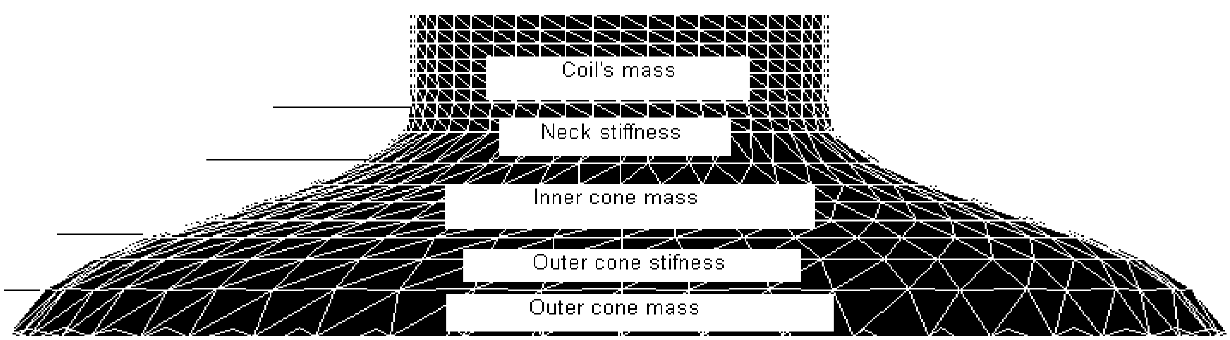


Figure 2: Strain of the moving assembly of a direct radiator after a static stretching process.

It is necessary to take into account that the large static stretching process exhibited in Figure 2 shows the neck stiffness and the outer cone stiffness as well, but these compliances act also when the moving assembly is moving in the small which is the aim of the paper. The importance of the neck's stiffness was observed in 1977 by J.M. Kates [11] who described the response of a loudspeaker with a mechanical filter (a spring) in this specific place. In 1983 A.J.M. Kaizer and W. Kopinga [12] patented a speaker with this device installed.

Modal analysis of thin and slender bodies by the finite element method is widely reported in literature; see for example the references [13] and [14]. Special interest has been given to axial modes by P. Larsen [15], despite the author not calling these modes "axial" and he does not explain why these modes are relevant.

From now on when we talk about the isolated moving assembly we will be referring to the one which is suspended by axial very soft suspensions. When performing linear modal analysis on an isolated moving assembly by finite element method there are two main axial modes. One is the axial mode in which the most strained spring is on the neck of the body assembly. This mode is depicted in Figure 3. Observe how the complete cone swings up and down bending on its inner edge or circumference, and observe how the former stretches and shortens in a circular fashion. From now on this mode will be called the neck axial mode. An equivalent bulk three degree of freedom system is inset in the figure. This equivalent system has the same coil mass and the same total cone mass. The equivalent system of the inset has a neck stiffness which is smaller than the cone stiffness near the rim. The other axial mode is the one at which the most strained spring is that of the outer cone near the rim. The mode is depicted in Figure 4. In the figure the sharing of masses of the inner cone and outer cone with respect to the total is taken arbitrarily. Observe in the figure how the outer cone of the isolated moving assembly "flaps" up and down. This mode will be called the "near rim" axial mode from now on. The equivalent bulk system depicted in the figure shows a motion in antiphase of the inner and outer cones, and a very small motion of the coil in antiphase with respect to the inner cone. If we swap the stiffness of the two axial springs of the moving assembly, then the first mode found will be the one of Figure 4 and the second will be the one depicted in Figure 3.

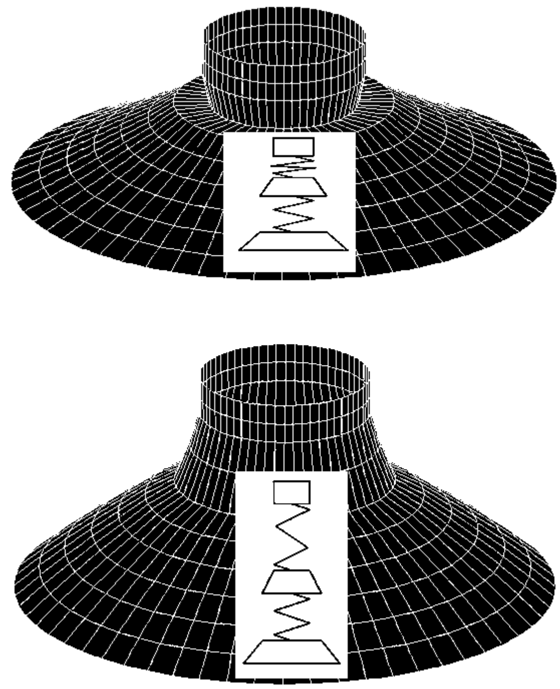


Figure 3: Neck axial mode of the moving assembly. Suspensions are omitted for clarity.

Only one of these masses (the coil) is forced by the electromagnetic force, the other two are linked only by their mechanic stiffness and associated damping. A more realistic approach to the speaker modelization in axial motion is a three degrees of freedom system with the inner and outer cone masses linked by a spring and damper, and the inner cone mass is linked to the coil by the neck stiffness (with its associated damper). Only the coil's mass is submitted to external forces. This subject will be addressed further with more detail.

2. 2. Main Suspension's Modes and Basic Interactions with the Axial Modes.

In 1998 D. Bie [16] reported a global analysis of the motion of a diaphragm and the associated suspension in a single suspension transducer, using both numeric analysis and experimental methods. In 2000 D. Henwood and his Colleagues [17] published an experimental work dealing with the same topic, where the interaction of a loudspeaker cone and the surround is shown evidently and the authors recognized it as of complex interpretation.

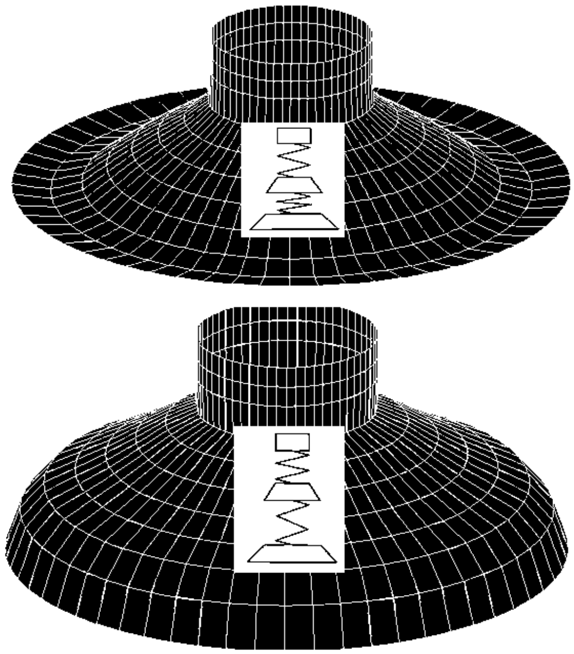


Figure 4: “Near rim” axial mode of the moving assembly, suspensions are omitted in the Figure.

There are specific suspensions modes to deal with. The main axi-symmetric motions of the suspension are the concentric-circular mode (circular accordion shape motion) and the in-phase-axial mode (flapping motion). The concentric-circular mode consists of concentric in-phase radial motions of all the elements that make up the suspension. The apparent suspension’s motion occurs, in the suspension’s plane, which is perpendicular to the transducer’s axis. Figure 5 illustrates this kind of motion for a half roll suspension. The concentric waves swing back and forth in concentric circles between the cone rim and the speaker frame. The concentric-circular modes are in spectral regions which are very dependent on the suspension shape and size. Often the surrounds of various rolls, have this type of mode at frequencies much higher than the half roll suspension. Figure 6 depicts the same type of mode in a two rolls suspension, the rest of the moving assembly is omitted on the figures for clarity. In both figures the suspension motion waving concentrically back and forth is visible.

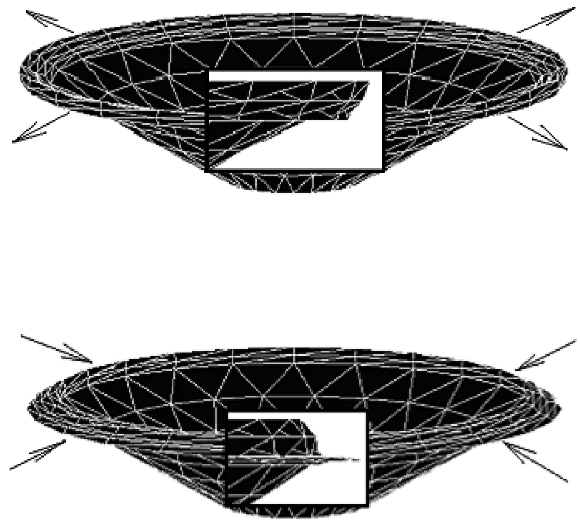


Figure 5: Suspension’s concentric-circular-mode (accordion shape). The suspension waves outwards and inwards radially. The model is a half roll surround.

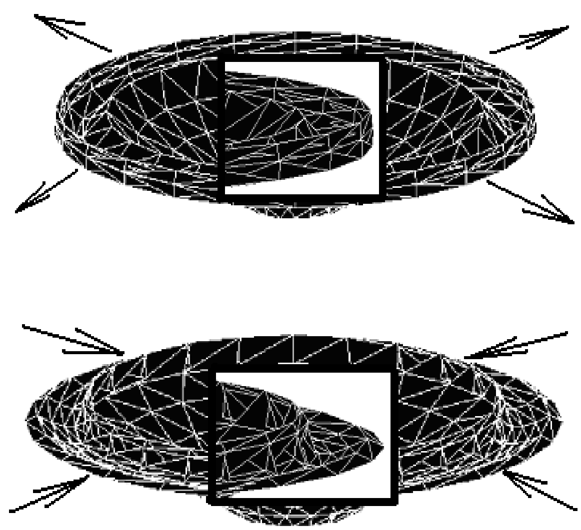
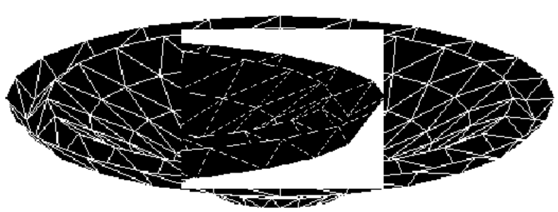


Figure 6: The same mode shape of Figure 5 (accordion shape) for a two roll surround.

While a flat suspension (or a real bellows suspension) is in the first axial-mode (flapping mode), the median suspension circumference is an antinode with all points moving axially in-phase. The outer suspension's circumference is a node (restrained by the frame) and the inner suspension's circumference is a node contour line or near a node contour line as well. Other axial-modes (flapping modes) exist in the suspension, for brevity they are not treated here, but they consist of suspension standing waves with an integer number of half waves between the inner and outer suspension's circumference. Figure 7 depicts the main suspension's axial-mode for a flat suspension, where the two external circular element rows belong to the suspension. The remaining elements belong to the cone. Figure 8 shows the same mode for a half roll surround. The natural frequency at which the concentric-circular-mode takes place, for the half roll suspension depicted in Figure 5, is lower than the natural frequency at which this suspension exhibits flapping. An almost pure flapping mode can be obtained for certain eigenvalues where the suspension moves in-phase and the rest of the body assembly yields at rest. This circumstance is accomplished frequently in several transducers.



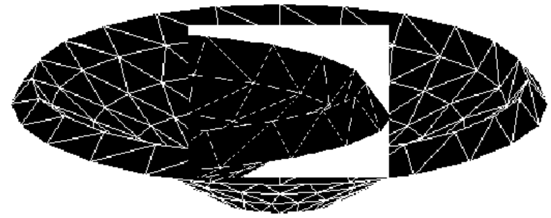
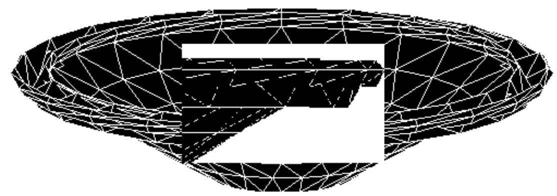


Figure 7: Axisymmetric suspension's mode which motion is in the transducer's axis (flapping-mode) for a flat surround. The surround has two rows of elements.



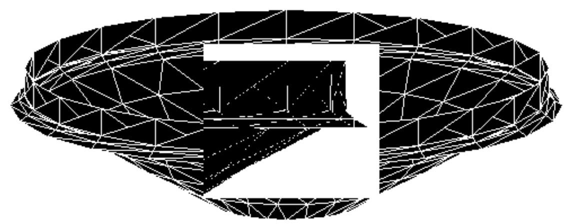


Figure 8: The same mode shape of Figure 7 (flapping mode) for a half roll surround.

We have dealt with axial modes of isolated moving assemblies and the basic suspension modes as well. However the real moving assemblies are suspended by two elastic devices. The spider may interact with the body assembly basically when the neck is swinging. In Figure 9 the same moving assembly that exhibited the neck's axial mode is depicted but with a spider attached to the body. It is evident in the figure that both bodies experience a synchronized motion. The natural frequency found is a bit lower than the one of a moving assembly suspended by an infinite soft suspension device. This indicates, basically, a small overweight applied to the neck. For simplicity the spider of the figure has been modelled flat, which gives reliable results for very elastic suspensions if the motions are in the small. Other spider shapes perform similarly to the one of the figure.

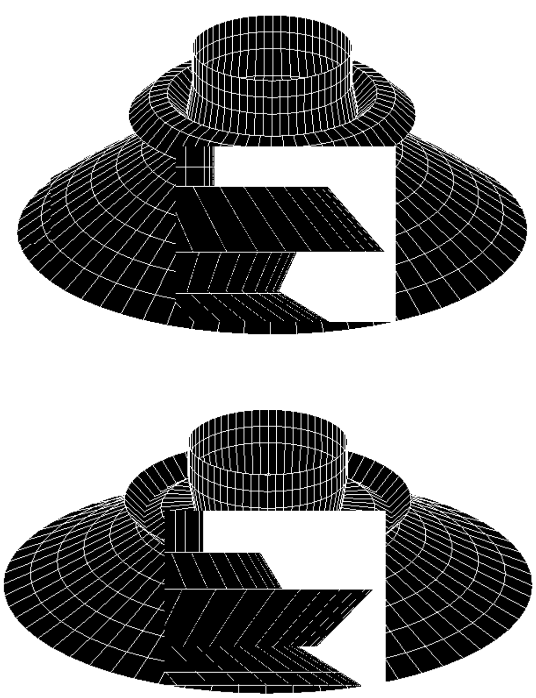


Figure 9: Interaction between the flapping mode of the spider and the neck of the moving assembly.

The surround has an even higher interaction with the moving assembly than the spider has. The surround may interact with the cone rim axial mode as well, lowering the natural frequency too. Figure 10 depicts the mode shape when both bodies are joined. As in Figure 7, the suspension is designed with two rows of elements in the model. Observe how the cone rim flaps up and down while the suspension is flapping as well. The inset of the figure shows clearly the edge of the cone rim.

Moreover, the suspension may interact with the moving assembly neck giving rise to a compound mode. This mode shape is depicted in Figure 11, were we can see how the neck's motion and the suspension flapping are linked. Observe in the figure the edge of the cone at the former side, marked with an arrow. These behaviours are for undamped linear systems, which is, obviously, a necessary simplification.

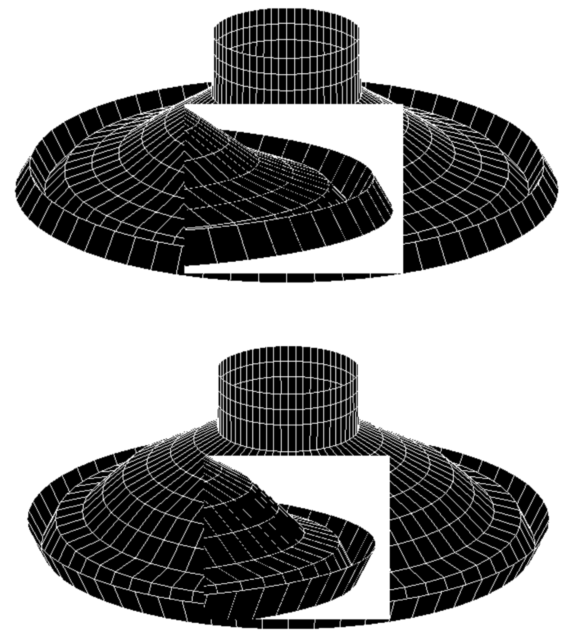
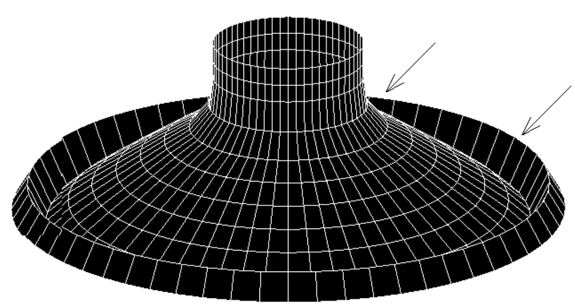


Figure 10: Interaction between the flapping mode of the surround and the rim of the moving assembly. The suspension is flat and has two rows of elements.



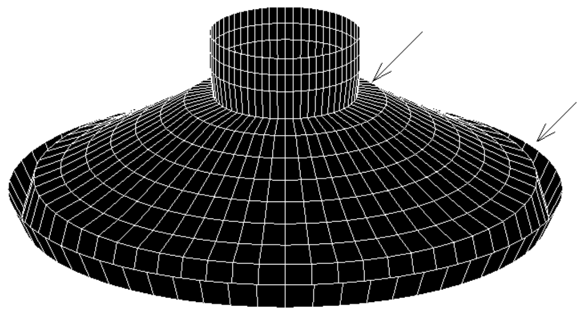


Figure 11: Interaction between the flapping mode of the surround and the neck of the moving assembly. The suspension is flat and has two rows of elements.

3. MEASUREMENTS ON A FIFTEEN INCH WOOFER AND NONLINEAR NORMAL MODES

In order to simplify the moving assembly dynamics and keep the moving assembly neck clear, the dust cap was not attached to the cone. For this purpose the experimental device was a 15 inch woofer which has a cone of 48 grams and a coil whose weight is 37.3 grams. The cone is paper made and has 12 circumferential shallow ribs on its body. The coil's diameter is 100 mm. The outer suspension is a two and a half roll cloth suspension.

The acoustic response of the chosen unit and the harmonic distortion of the second and third order are depicted in Figure 12, we will return to this figure later on in the text. The vectorial impedance curve is depicted in Figure 13, where besides the main resonant frequency a small hump around 604 Hz is visible. Amplitude oscillations below the main resonance are due to the fact that the sine sweep does not cover the very low frequencies.

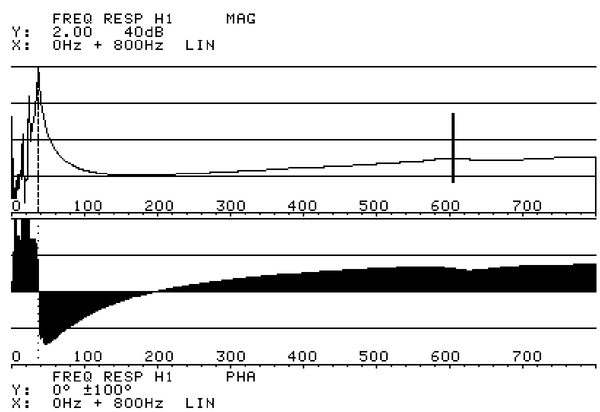


Figure 13: Vectorial motional impedance of the tested unit.

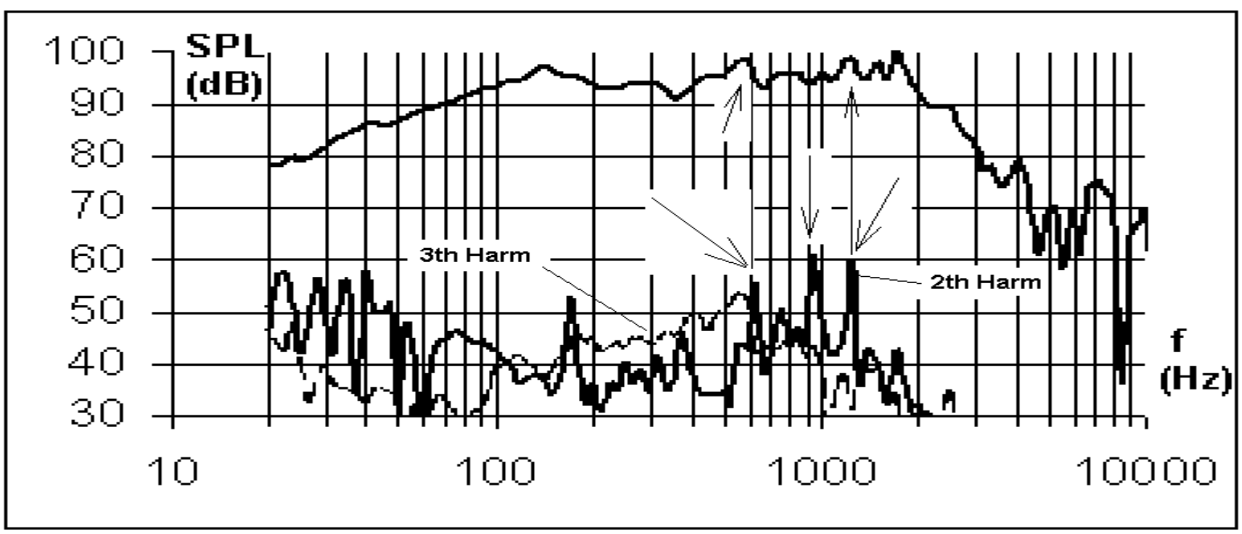


Figure 12: Acoustic free field response of the 15 inch woofer and 2nd and 3rd order harmonic distortion of the unit.

3. 1. Acceleration Free Response of the Moving Assembly

In order to perform free response measurements at medium frequencies, very small accelerometers were glued to the cone. The accelerometer with the soldered cable has a total weight of only 0.4 grams. For this purpose a pair of bimorph ceramic elements, working in bending, were glued to the cone at different ribs heights. The accelerometers were installed for each measurement in the same cone rib, at the end of diametral lines, providing weight balance to the measured cone. Figure 14 depicts a moving assembly and the measurement points A and B. The pair of measuring points A, which belong to the same diameter and rib, are 27 mm away from the inward cone rim. This site can be assigned to the outer cone. The pair of measuring points B, which belong to the same diameter and rib, are 67 mm away from the inward cone rim. This site can be assigned to the inner cone. The free response of the moving assembly was tested applying axial forces to the isolated body at the coil at its free side (the bottom of the photograph). For this purpose a small lightweight wood cross bar was attached diametrically to the coil at the free side; this provides a simple no invasive form to apply axial forces to the whole isolated moving assembly.

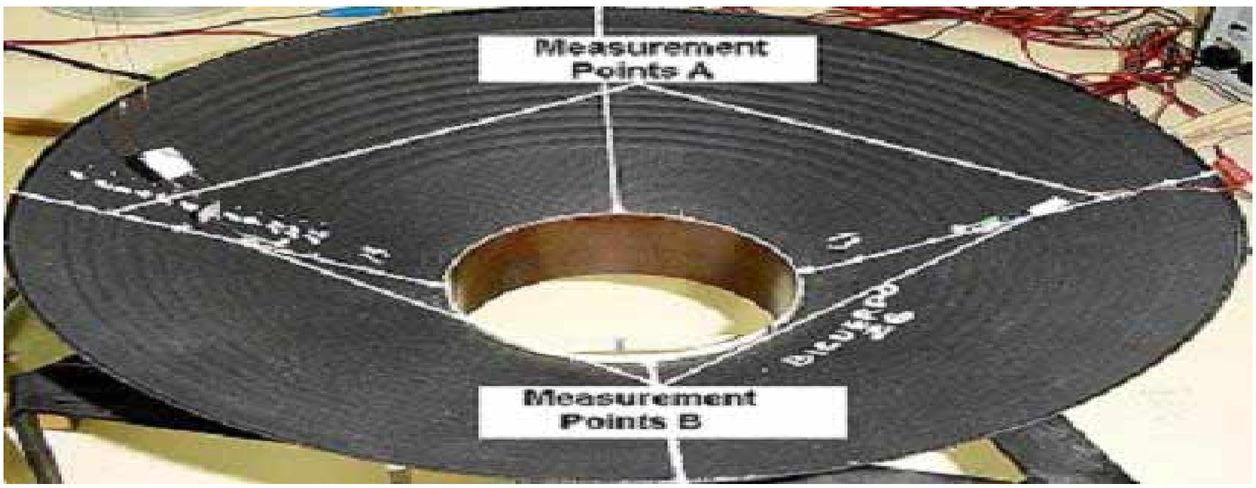


Figure 14: Measurement sites A and B for a pair of accelerometers on each site. Measurement site A corresponds to the outer cone, site B is in the inner cone. The moving assembly is suspended by its spider.

The responses found are level dependent, they depend on the initial conditions, and the spectral signature depicts clear nonlinear shapes [22]. Responses when the input force was applied compressing the moving assembly springs are different than those found for input forces applied extending the springs.

In order to find results as close as possible to the complete unit, measurements were taken testing the complete moving assembly with its inner and outer suspensions. The results obtained for inputs that tend to extend the moving assembly springs have much better spectral appearance (linearity) than those that shorten them. This is reasonable because when the force compresses the springs, most of the applied energy is absorbed by the first axial normal mode of the speaker (which is the regular speaker resonance). This is due to the higher elasticity of the suspensions than the neck and the near rim springs have. When the lower mode is highly excited the higher order modes are much less activated or excited.

Figure 15 depicts the spectral simultaneous responses measured by two accelerometers at measurement points A. Figure 16 depicts the spectral simultaneous responses measured by two accelerometers at site B. The dynamic range of these graphics is 40 dB as indicated in the display set up. Despite the full scale and the graphics cursors being in millivolts their magnitudes are calibrated in dB with respect to a common arbitrary reference.

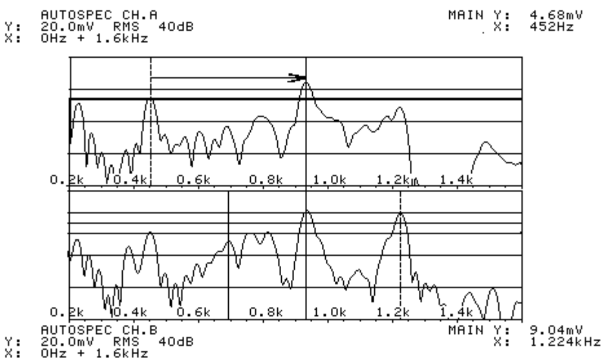


Figure 15: Acceleration responses of the two light weight accelerometers on site A (outer cone).

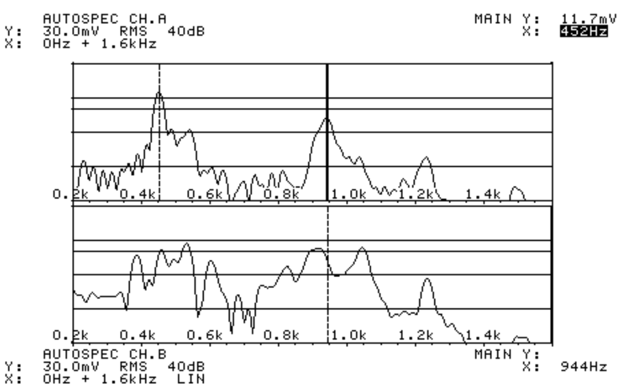


Figure 16: Acceleration responses of the two light weight accelerometers on site B.

We conjecture that these responses are clearly non linear responses. Observe how in the figures the accelerometer outputs do not deliver the same signals. The simultaneous spectra are rather different, both for measurement site A and B. The outer cone responds with a spectrum which is much more complex than the inner cone. This is reasonable because the inner cone is closer to the input force than the outer cone, and the outer suspension adds additional damping to the outer cone. The inner cone (measurement points B and Figure 16) respond with peaks at 944 Hz and 452 Hz approximately. Observe how these peaks have closeness to the ratio 2:1. The proximity of these frequencies to the 2:1 ratio is close enough for a high damped system which is the moving assembly materials. When the free response of a system delivers natural frequencies close to the ratio 2:1 those systems are said to have an internal resonance of the type two-to-one, see these definitions in [21]. These frequencies are probably nonlinear normal frequencies of the nonlinear normal axial modes. In spite of the response in the outer cone area, (measurement points A, and Figure 15), being more complex than the inner cone area both peak at 930 Hz and 452 Hz and these are clearly visible as well. However there are new raising evolving spectral regions at 1220 Hz and 680 Hz approximately, which has the same rough frequency ratio of 2:1. The peak of 1224 Hz seams to be the second axial mode. The proximity to the outer suspension, which has several modes as a physic subsystem as we saw in paragraph 2.2, must be taken into account. Observe that the second harmonic acoustic distortion curve H2 (Figure 12) has these two peaks at 950 Hz and 1250 Hz respectively, and the acoustic frequency response has important peaks at 1000 Hz and 1250 Hz as well. Conjectures are common on the nonlinear field; see for example [24], [25] and [26]. At high frequencies conjectures in modal analysis are usual as well; see for example reference [37] where the author says that *"much experimental work has gone unpublished, largely due to tenacious questions regarding statistical reliabilities, reproducibilities and fluctuations"*. In references [22] and [23] for example, dealing with nonlinearities, the word *signature* is used to explain qualitative results. Moreover, when concerned with engineering results, in highly damped nonlinear devices of several degrees of freedom, the necessity of conjecture is even higher.

3. 2. Nonlinear Normal Modes (NNM) on the Moving Assemblies

A system which responds as Figure 15 and 16 depicted, obviously, does not fulfill the superposition principle, and therefore the definition of a linear normal mode. In accordance with the author Rosenberg [18] normal mode vibrations of an autonomous (free) multi degree of freedom system are *"the vibrations-in-unison"* of the particles of this system. This implies that while the system is in a normal mode all masses of the body move in synchronization, attaining their maxima, equilibria and minima at the same instants of time. If the system we are dealing with is linear, the motion of all the masses follows a harmonic function. For a linear system undergoing a normal mode, as the response is harmonic, the relation of amplitudes between two elected points of the system has a constant value. This is due to the fact that each motion is a harmonic function that can change only in the amplitude. In literature those modes are called similar normal modes.

If a vibrating motion of a body is in unison it does not describe harmonic functions in all masses. In contrast, the relation between the amplitudes of two points, elected randomly, is not constant. In this case the normal mode is called non-similar. Many nonlinear normal modes (NNM) are non-similar. Reference [19] explains that in symmetric two degrees of freedom systems and homogeneous systems of many degrees of freedom, the modes are similar, independent of the amount of nonlinearity and independent of the amplitude of the responses as well.

An important fact is that in a linear system the number of eigenvalues equals the number of degrees of freedom, but this assertion does not hold for strong nonlinear systems, where, generally, the number of eigenvalues is larger than the number of degrees of freedom. This point is evident in the measurements made in the moving assemblies tested.

The theory about how to solve nonlinear systems is vast and beyond the scope of this paper, see for example the books of A. Nayfeh [20] and [21]. Here in this paper the interest is focused on the interpretation of the experimental results. Moreover, for example, experimental results obtained in moving assemblies [22] when they are observed and analyzed as nonlinear normal modes (NNM) may have interpretation, although when observed and analyzed using the linear theory they do not.

The study of the mathematical structure of NNM has acquired an important advance, but the practical experiences have been done, basically, with simple devices and in laboratory conditions. Most of the experiments are done with bars, strings, plates and simple frame structures. For example, the paper of [23] deals with the transference of energy through the modes and reports real measurements in a beam. In this paper the author uses the word “signature” for a modal interaction, as the author of the reference [22] does for the spectral shape for moving assemblies. Much attention is paid to damped systems by A. Vakakis and his colleagues [24].

The phenomena reported in [24] are very significant, since a necessary condition for the execution of certain energy transference processes is that the system must have an important degree of damping. This is the case of the moving assembly of the direct radiation loudspeakers. The authors of [20] explain the paradoxical fact that: *the energy dependence of the free synchronous periodic solutions (NNM) of the undamped, unforced (autonomous) system governs, in essence, the energy pumping properties of the corresponding damped and forced system.* This circumstance is important for loudspeaker dynamics as well, although the moving assembly responds in a nonlinear fashion, the *underlying* linear system governs the performance of the device. This justifies the use of the linear modal analysis by the FEM, justifies to apply linear ordinary differential equations (ODE) models, and suggest perform real measurements of free responses in the moving assemblies as well.

Because of the amplitude dependence of the response, the author A. Vakakis [24] graphically represents the frequencies of the NNM (obtained when the system under test is subjected to free response) as a function of the energy of the motion, which is the physical energy of the system for the corresponding NNM under consideration. On the other hand, the dynamics of an oscillator close to a NNM is not “smooth” and “totally predictable”. The motions on the configuration space take place with complicated trajectories, and occur in stochastic layers.

The case of low energy motions (small signals) may be deceiving with respect to the case of large energies, because for the moving assembly materials and geometries; motions of low energies are not smooth and have low predictability as well. As it is explained in [19] these systems have extreme sensitivity to initial conditions.

In the speakers field the authors F..M. Murray y H..M. Durbin, [32] called certain peaks or clustered peaks they found while measuring motional impedance of compression drivers *activity*. The authors also reported the shift or miss of some high frequency modes for a bad transducer of the same type. The shift or miss of the high frequency mode caused a sensitivity loss in the corresponding frequency range and a rejection of the unit by the quality control staff. It seams that these *activities* and the weak stability of a mode can be classified as nonlinear normal modes of the moving assemblies of the tested compression drivers.

3. 3. Displacement Free Response of the Isolated Moving Assembly

Displacement responses were done in an isolated moving assembly suspending it by the spider, which is the less influential suspension on the whole response. The excitation signal was a short axial impulse applied to the coil at the free side by means of the wooden cross bar. The measurements were done following the criteria used with the accelerometers, at the measurement points A depicted on Figure 14. For brevity only the measurements in the outer cone are shown.

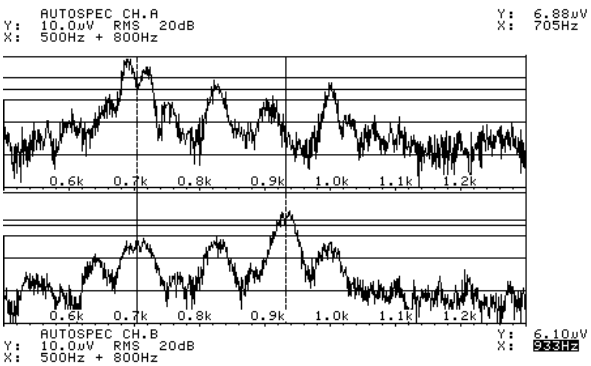


Figure 17: Displacement free responses of the moving assembly measured at site A (outer cone). The outer suspension is attached to the moving assembly.

Again, the sample exhibited different behavior when force was applied stretching or shortening the moving assembly neck. Figure 17 illustrates one of the responses for a pulse that stretches the neck. The bottom graphics enhance the founded peak of 933 Hz with several jointed peaks. On the contrary, the other displacement transducer (upper graphic), delivered a spectrum which enhances a split peak around 705 Hz. This peak may be the evolution of the one found when we measure with accelerometers, formed by the pair of nonlinear normal frequencies of 680 Hz -1220 Hz. Notice that the tested moving assembly whose results were those of Figure 15 was suspended only by its spider.

As expected, and in coincidence with Vakakis [19], the response was sensitive to the excitation level. This gives an additional complexity to the interpretation of the results.

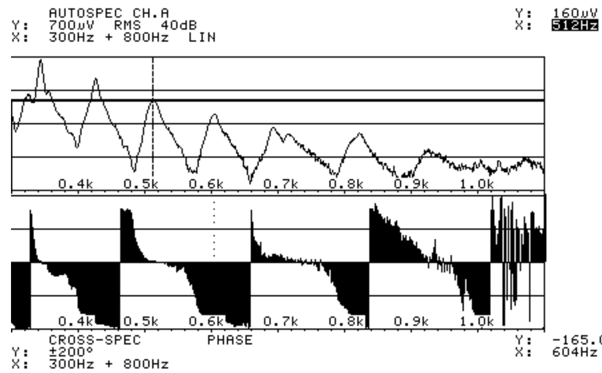


Figure 18: Lock-in of the radial modes of the moving assembly when submitted to an axial impulse.

When we measure the free response of the isolated moving assembly in the low frequency range for certain input force levels, and for a force direction which shortens the neck of the test specimen, we can see that the axial modes may lock-in the bending modes (called radial in paragraph 2) of the cone. Figure 18 illustrates the response, showing at the top the amplitude of one displacement transducer and the bottom the phase between both displacement transducers. Observe the spectral sequence of modes showing alternatively a phase of 0 and π radians, which corresponds to bending modes (radial modes) on the cone, whose mode shapes (see Figure 1) have even and odd sides respectively. The responses delivered a high spectral coherence between both measurement points and even better spectral shape (smooth) than the response for a radial pulse applied directly to the cone. For brevity this is not illustrated here.

3. 4. Forced Response of the Woofer

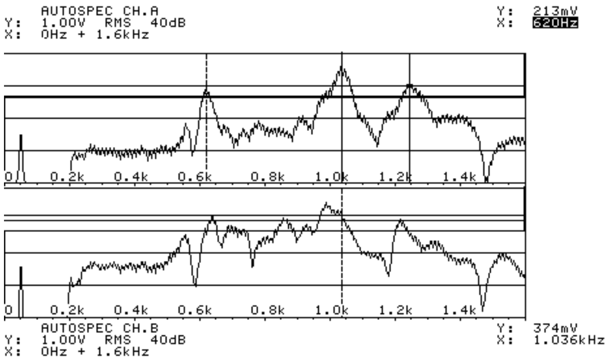


Figure 19: Forced acceleration responses of the moving assembly measured on site B.

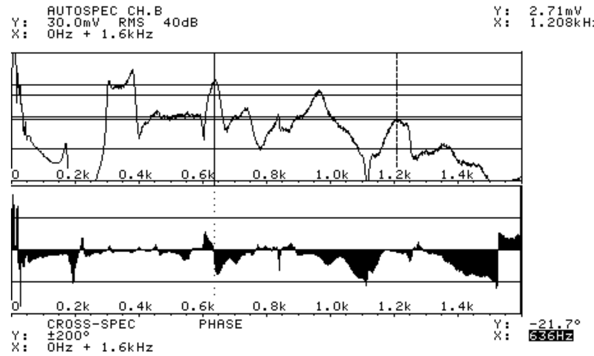


Figure 20: Forced vectorial displacement response of the moving assembly measured on site A (outer cone).

Despite most of the literature which deals with non linear normal modes being addressed to the free response of the systems (see for example [23], [24], [25], [26]), it is convenient to know the forced response of them as well. In order to complete the measurable responses of the moving assembly, the forced response, which is essential, was measured. The unit was measured while driven by a voltage sine sweep. Because the tested sample has one of the outer suspension modes seen before, close to one of the axial modes; the acceleration response at the measurement points A, is more complex than the inner cone response (measurement points B). For this reason the forced response of the woofer at measurement points B is shown. As in previous figures, Figure 19 shows the forced acceleration response, in dB but referred to an arbitrary level. In the forced response the set of frequencies 620Hz-1240Hz is clearly depicted and the frequency of 1036 Hz may correspond to the free response of 930 Hz as seen before.

3. 4. 1. Displacement Forced Response of the Cone

The displacement measurements performed at the measurement points A of the cone gives the results depicted in Figure 20. The figure contains the magnitude response of one displacement transducer and the phase between both measuring devices for a sine sweep on the woofer in the range 300 Hz – 1600 Hz. In the Figure the most outstanding peaks are 636 Hz, 964 Hz and 1208 Hz. These peaks correlate well with those found by accelerometers. The figure shows a high phase mismatch around 630 Hz, this is mainly due to interference or interaction the cone has with the main suspension flapping mode, which is spectrally close, and that couples in our sample with the axial mode. This point is not treated in detail for reasons of brevity. The small peak and phase deviation at 460 Hz is due to the rocking mode around the neck which the moving assembly has. This mode will be shown later on. Phase deviation in the graph is not too relevant because the loudspeaker is working forced by the force exerted on the coil, and the moving assembly is not in free motion.

3. 4. 2. Coil's Forced Response

The displacement of the free side of the coil was measured on a sample which was identical to the one used for forced responses. The sample for this measurement has a wider air gap and four holes on the bottom magnetic plate, to provide access to the free side of the coil. A lightweight, narrow and stiff cardboard ring was glued to the coil at the free side as a light reflector.

Figure 21: Measurement of the coil motion using drilled holes on the magnetic back plate in a loudspeaker which has a wider air gap.

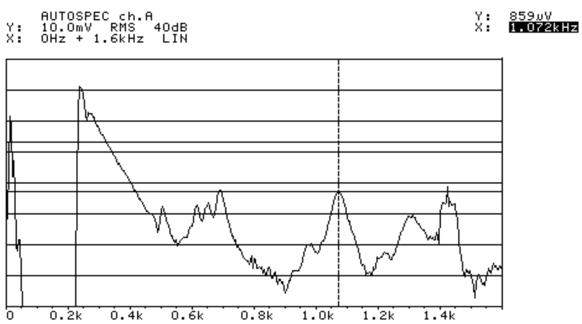


Figure 22: Forced displacement response of the coil measured with the set up of figure 21.

A detail of the measuring set up and the tested sample is depicted in Figure 21. The displacement response is illustrated in Figure 22. Observe at low frequencies the straight line falling with frequency and at medium and medium-high frequencies the humps the figure depicts. Observe the peaked region of 612 Hz- 688 Hz and two ranges more, one centered at the cursor (1072 Hz) and the other of wide band with two peaks at 1300 Hz and 1400 Hz. Observe the response has clear nonlinear signature and note how 650 Hz and 1300 Hz has an important displacement response. These peaks have an increase in frequency respect to those found measuring at the cone. Despite the Bl of the modified tested sample having diminished in respect to a standard unit, this does not significantly change the outstanding frequencies of the response, and the measurements are valid for the targeted purposes.

4. A LINEAR MODEL OF THE SPEAKER BASED ON THE AXIAL MODES

Despite the moving assembly having an evident nonlinear behaviour; the response will be controlled, mainly in practice, by the corresponding underlying linear part. A model of the speaker that assumes that the whole moving assembly mass is bulk or concentrated in the coil does not correspond well with reality, because the neck and outer cone compliances are important in the upper part of the transducer frequency response. At high frequencies the cone mass is linked mechanically with the coil but not electromagnetically as the coil's mass is. In reference [27] the author Friedland calls the system which is considered to have less degrees of freedom than it actually has, a *physical uncontrollable system*. A speaker model with a single mass can be seen, for example, in the work of A Bright [24].

Following the axial modelling of Figures 2, 3 and 4, applying the force to the coil mass, and adding the equation which supplies the coil intensity, the global moving assembly can be modelled as follows:

$$m_{coil} * x_{coil}'' = - c_1 * x_{coil}' - k_{sp} * x_{coil} - k_{neck} *(x_{coil} - x_{i\_c}) + Bl*i \qquad (1)$$

$$m_{i\_c} * x_{i\_c}'' = - c_2 * x_{i\_c}' - k_{neck} *(x_{i\_c} - x_{coil}) - k_{o\_c} *(x_{i\_c} - x_{o\_c}) \qquad (2)$$

$$m_{o\_c} * x_{o\_c}'' = - c_3 * x_{o\_c}' - k_{o\_c} (x_{o\_c} - x_{i\_c}) - k_{susp}\, x_{o\_c} \qquad (3)$$

$$i = (V*Cos(2*\pi *f*t) - Bl * x_{coil}') / (R+2* \pi *f*L) \qquad (4)$$

Being : x_{coil}, x_{i_c}, x_{o_c} the displacements of the coil, the inner cone, and the outer cone respectively. The primes denote differentiation with respect to time.

m_{coil}, m_{i_c}, m_{o_c}, the coil mass, the inner cone mass and the outer cone mass respectively.

k_{sp} and k_{susp} are the stiffness of the spider and the stiffness of the suspension.

k_{neck}, and k_{o_c} are the stiffness of the neck and the stiffness of the outer cone.

c_1, c_2 and c_3 the damping associated with the elements of the model.

Bl is the transducer conversion factor (or the transduction coefficient).

R is the electric resistance, V is the voltage, i is the electric intensity, f is the frequency and t is time.

For simplicity here the inductance L is assumed as a constant.

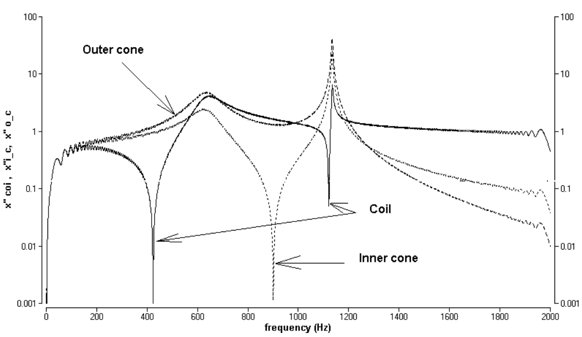


Figure 23: Acceleration responses of the three masses of the modelled moving assembly for the values given in the text for a neck 20% softer than the outer cone.

The equations system (1), (2), (3) and (4) has been solved for a speaker which has the following parameters and applied voltage:

m_{coil}	m_{i_c}	m_{o_c}	k_{o_c}	k_{neck}	Bl factor	Volt
37 g	35 g	15 g	479000 N / m	0.8 * k_{o_c}	22 N / A	1 V

In this case the neck stiffness is assumed to be 20% softer than the outer cone spring stiffness. The stiffness value k_{o_c} is estimated, based on measurements, but unfortunately the measurements are inaccurate. The numeric solution is depicted in Figure 23 for the case of the three damping factors equal to 0.1. In the figure the acceleration spectra is depicted for the three split masses of the moving assembly. Observe the two upper poles of the system (the first pole is very much damped). Observe how the maximum acceleration values at low frequencies are obtained by the outer cone, and the maximum acceleration values at high frequencies are obtained by the coil. Notice, the two zeros the coil has and the single zero the inner cone has.

The spectral shapes of the figure correlated reasonably well with the experimental acceleration forced responses illustrated in Figure 19, and with the displacement forced responses of Figures 20 and 22. The presence of the double zero in the coil response and the single zero of the inner cone response illustrated in Figure 23 justifies the experimental results obtained close to the coil which have better spectral definition than those obtained at the outer cone side. The real moving assembly exhibits nonlinear normal modes and some of the spectral peaks appear in the graphic in a pair, as the 620 Hz and 1240 Hz of Figure 19. It is obvious that the linear three degrees of freedom has three resonances and the real nonlinear moving assembly has its intrinsic spectral complexity. However the underlying linear system is visible in Figure 19.

If we assume a neck 20% harder than the outer cone spring, the forced spectral response is the one illustrated in Figure 24. Despite two of the upper poles shifted up in the spectrum, observe that the second resonance (of the three) is the most sensible to this stiffness change. In Figures 23 and 24 we can see that due to the stiffness increase of the neck, the zeroes have shifted up in the spectrum as well, but the zero of the inner cone almost remains at the same place. The spectral shift of the poles and the zeros can be used as a tool for mode finding, when using experimental techniques, but it is difficult in practice due to the spectral complexity of the NNM modes of the real moving assemblies. The spectral shift of the zeros is hardly detected due to the large lack of signal in these spectral regions.

A model that splits the cone in three masses was proposed by G. Pellerin and his colleagues [29], however this model links the electromagnetic circuit with each individual mass, and these masses do not belong to a system of coupled masses by its neighbour stiffness (the three degrees of freedom system). A model which considers several degrees of freedom on piezoelectric transducers is reported in reference [30].

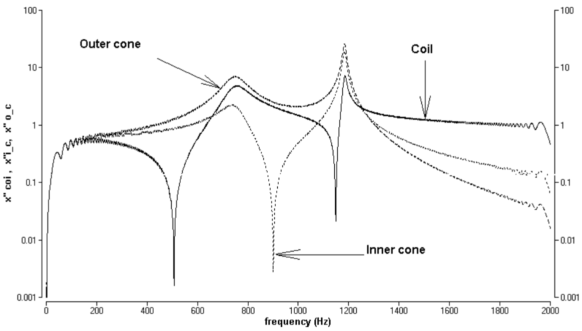
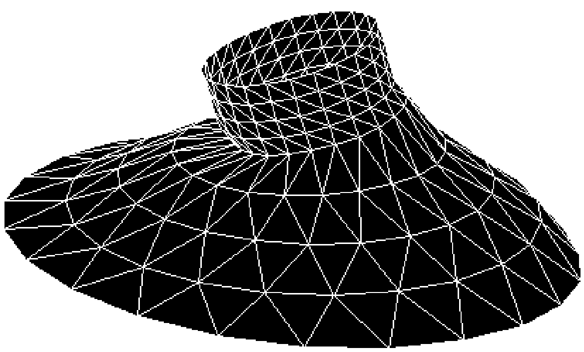


Figure 24: Acceleration responses of the three masses of the modelled moving assembly for the values given in the text for an outer cone 20% softer than the neck.

5. OTHER MOVING ASSEMBLY SIGNIFICANT MODES, AND LOCAL MODES.

Beside the main axial mode there are some significant modes that the moving assembly has. One of these modes is the 3D bending or rocking of the two main masses (cone and voice coil) around the neck, which performs as a spring. The neck-spring exhibits global-bending flexibility.



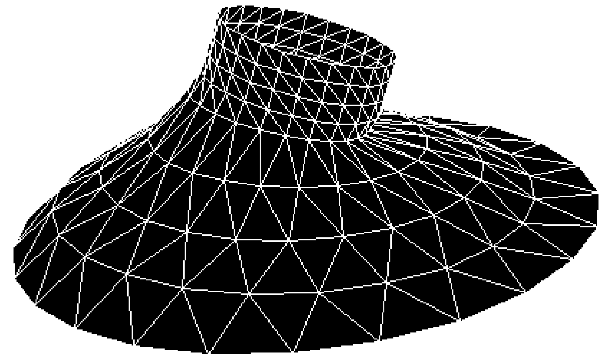


Figure 25: One of the two orthogonal components of the motion of the moving assembly while it is performing a rocking neck mode. Suspensions are omitted in the figure.

Figure 25 depicts one of the two orthogonal components of the bending global mode around the neck, were the suspensions are hidden for clarity. The real motion has two components, and this motion can be described as follows: Any point of the free edge of the voice coil describes a circumference which belongs to the base of a virtual cone, with the apex at the side of the elastic centre of the neck. Any point of the cone rim describes a circumference which belongs to the base of a virtual cone with the apex at the side of the elastic centre of the neck as well. The motion of the bended axis of the moving assembly describes two cones joined by their apex. The motion is alike two spinning tops moving in a free space with precession (but not spinning), restrained axially by their apex. The spinning tops (cone and voice coil) are mounted face to face and joined by their apex. This mode is a rocking mode similar to the well known regular rocking modes [31], but instead of a single inertia (the global moving assembly mass) related with two suspensions, it is caused by the two main inertias (voice coil and cone) acting over the neck bending stiffness. This mode can be called bending with rotation around the transducer axis by the neck, or rocking mode turning (or rotating) by the neck. This mode practically does not radiate sound. Due to the materials damping and potential nonlinearities the mode can influence other modes close to it in the spectrum. On the other hand, this mode does not produce any control force on the voice coil because its motion is not axial, and the mode is not restrained by the electromagnetic forces.

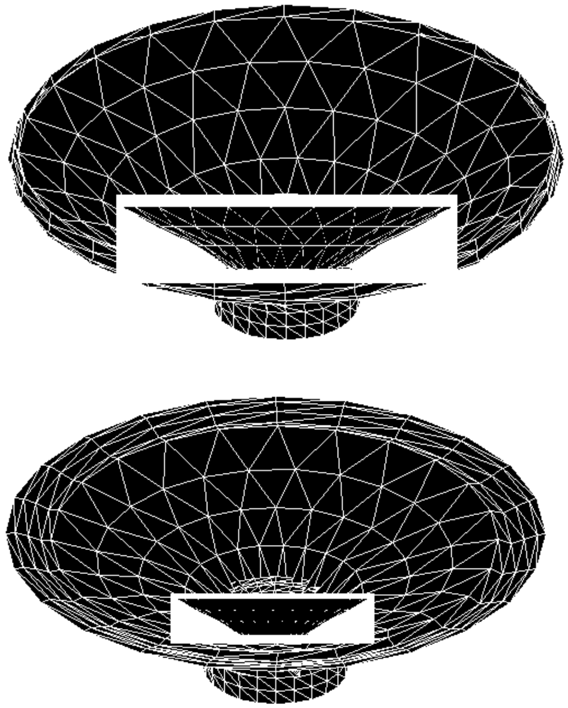


Figure 26: Main extensional mode of the cone. This mode is commonly called "Cone breathing mode".

The high frequency roll-off of the acoustic response of the tested woofer, starts after a relevant peak of 1700 Hz. The modal analysis by the FEM provides a mode of the cone called the extensional mode. While the moving assembly is in this mode the cone is moving with all particles in phase with the motion in the cone plane. This extensional mode is often called breathing mode [10]. In a real loudspeaker while the cone is vibrating on its extensional mode, most of the rest of the moving assembly and suspensions adapt their shape to the cone in-plane motion. The cone extensional mode is depicted in Figure 26. While the cone is moving in this mode it stretches and shortens periodically as it is heated and cooled periodically. The extensional mode is important in all kinds of transducers because the high coherence of the motion [38] and consequently the high acoustic radiation, see references [1] and [2]. To verify if the peak of 1700 Hz corresponds with the extensional linear normal mode is beyond the purpose of this paper.

The suspension's modes depicted in figures 5, 6, 7 and 8, are local modes of the loudspeaker because the vibration energy of the moving assembly is confined to the suspension while the speaker is moving on this mode shape. These modes are generally undesired by the loudspeaker designer, because of their uncontrolled condition. Confined vibrational energy has been known in acoustics for many years. T. D. Rossing reported this confinement in the mridanga Indian drums [33], in which most of the vibration energy is confined to the loaded portion of the drumhead. The confinement of vibration energy in vibrating structures has been studied deeply both for linear and nonlinear systems for slender periodic structures. In this field the phenomenon is called localization, see for example [34], [35] and [19]. One intentional form of building up one or various local modes, which significantly extend the usable frequency range in loudspeakers, is the use of a double cone or a whizzer.

The performance of this transducer has been explained by J.S. Stewart [36]; in this paper the author explains that most of the whizzer radiation (and interference) comes from the walls or *"bell"* of the whizzer. The author also says that the functional mechanism of the whizzer is not particularly well understood.

A particular case of a local axial mode is the one developed in the whizzer. When a double cone speaker moves in this particular mode shape the delivered acoustic output is high due to the high coherence [38] the mode has. Figure 27 depicts the local axial mode of the whizzer of a double cone speaker. The model depicts only one suspension that is flat for simplicity. Observe that in this mode shape the suspension and the remaining part of the moving assembly are at rest while the inner cone is developing its neck's axial mode. Because the inner cone does not have outer suspension the whizzer has the outer edge free, and the local axial mode and the extensional mode are able to develop high temporal and spatial coherence. This axial mode and the extensional (breath) mode of the inner cone (not shown for brevity), which both occur at high frequencies, are the main cause of the extension of the frequency response of the transducer.

The inconvenience of this double cone system is reported by J. S. Stewart in [36] *its "choppy response" and "terrible off-axis characteristics"*. These drawbacks are because extra modes have been added to the moving assembly in respect to the same moving assembly with only its outer cone. Some of these modes caused cancellations, interactions and irregular motions of the whole moving assembly.

Figure 28 depicts a mode at which the air pumped by one of the cones is absorbed by the other because of its counterphase. Figure 29 depicts one of the two orthogonal components of a local rocking mode around the neck of the inner cone.

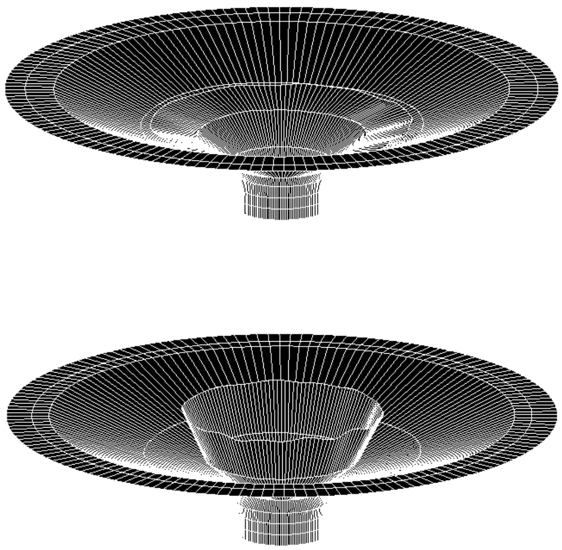


Figure 27: Local axial mode of the whizzer of a double cone loudspeaker.

This mode, which is equivalent to the one in Figure 25, in a single cone speaker, which has nonlinearities on the moving assembly, may disturb or interact with the main radiating modes of the transducer. The mode can be better defined meshing the model finer, but the mode shape depicted in the figure is clear enough. The frequency range at which these disturbing modes appear are, as the spectra shown in [36], lower and much lower than the whizzer local axial mode and the whizzer extensional mode. It is obvious that a model with three degrees of freedom as proposed in paragraph 4 will not be accurate enough for a double cone moving assembly. It is obvious as well, that these modes treated here are some of the underlying linear modes the double cone speaker has and the real speaker will obey the nonlinear physical laws.

6. CONCLUSIONS

Prior to obtaining the acoustic response of a direct acoustic radiator it is convenient to dynamically test the moving assembly verifying if it satisfies the performance targeted on the project specification. The modal analysis of the moving assembly and the suspension by the FEM provides the main linear normal modes of the woofers. The axial modes of the complete moving assembly are important eigenvalues and eigenvectors, and have been treated in some detail. The elasticity of the cone near the rim may interact with the suspension giving compound modes (of an axial mode and a suspension's mode) which make it even more difficult to analyze the moving assembly, and may be a cause of significant influence on the acoustic response.

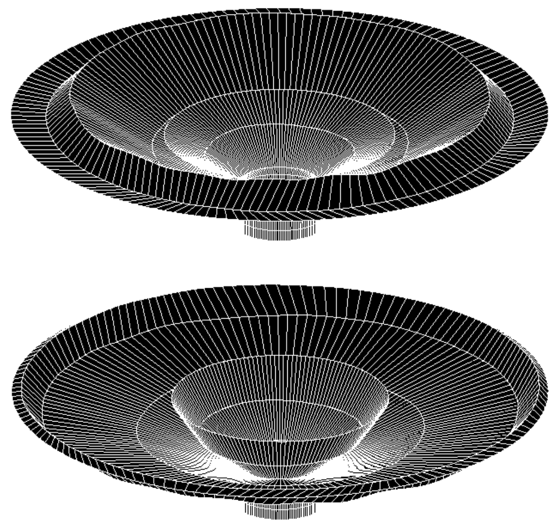


Figure 28: Axial mode of the double cone speaker at which both cones move in antiphase. The surround has two rows of elements.

The paper has reviewed significant modes which the moving assembly has, including local suspension's modes and relevant modes of double cone moving assemblies including the whizzer's local axial mode and the whizzer's extensional mode.

By standard laboratory instrumentation, and by traditional measuring techniques, the main natural frequencies of the moving assembly can be found at the low and medium frequency range of the woofer. The acoustic response, impedance function, free response and forced response data, correlate reasonably well. The acceleration and displacement free response of the moving assembly gives the natural frequencies that will be found in the rest of forced measurements responses, including the acoustic response and the harmonic distortion. On the other hand, phase measurement is an inexpensive and valuable tool for mode classification.

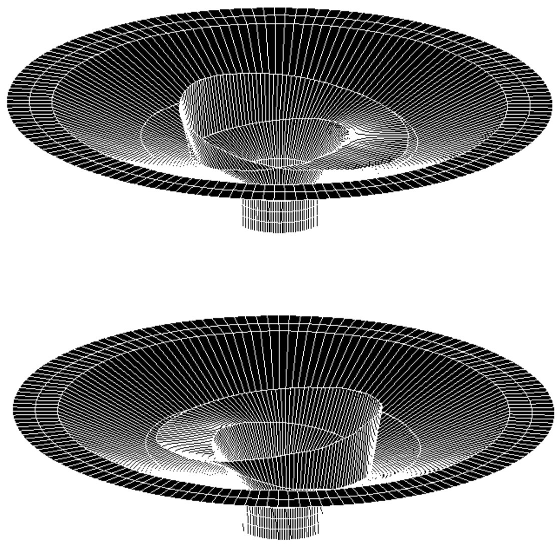


Figure 29: One of the two orthogonal components of the rocking mode around the neck of the inner cone of the double cone speaker (a local mode).

The experiences show evidence that the modes of the moving assembly are nonlinear normal modes (NNM), which have specific treatment in literature. For forced high damping nonlinear systems such as the loudspeakers moving assemblies, the study of its linear counterpart is essential because its dynamics are governed by the underlying Hamiltonian (undamped) unforced system. The tested woofer seems to exhibit quadratic nonlinearities for its motion in the small. These quadratic nonlinearities must be analyzed carefully in order to give physical sense to the results. The frequencies found by experimental procedures are repeated in free and forced response, in acceleration and displacement measurements, in motional impedance, and finally in pressure response and harmonic distortion measurements as well. The main practical drawback is the closeness of a subsystem natural frequency (for example the suspension) to one basic NNM; this circumstance obscures the experimental results.

It seems convenient to treat the moving assembly elements (cone, voice coil and suspensions) of the direct acoustic radiator as a three degrees of freedom vibrating structure, with electric control only applied to the voice coil. Despite the fact that using a three degree of freedom model may be a simplification, based on the axial modes; those models based on the moving assembly as a single mass are oversimplified [6]. The apparent paradox that some high renowned loudspeakers have high nonlinear moving assemblies should be accepted by those who are not familiar with nonlinearities. Due to the use of high transducer conversion factors (Bl), the moving assemblies are driven at regimes of high energies where the nonlinearities due to the moving assemblies themselves are less evident [37] and [38].

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