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Target Modes in Moving Assemblies of Compression Drivers and other Speakers

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ABSTRACT

The paper deals how to find the important modes in the moving assembly of compression drivers and other loudspeakers. Dynamic importance is an essential tool for those who work on modal analysis of systems with many degrees of freedom and complex structures. The important modes calculation or measurement in moving assemblies is an objective (absolute) method to find the relevant modes which acts on the dynamics of these transducers. Paper deals about axial modes and breath modes which are basic for loudspeakers. The model generalized masses and the participation factors are useful tools to find the moving assemblies important modes (target modes). The strain energy of the moving assembly, which represents the amount of available potential energy, is essential as well.

1. INTRODUCTION AND PURPOSE

Target modes are those mode shapes that are determined to be dynamically important using some definition [1]. This concept has been widely used in various fields as the aeronautics, space engineering and in seismic engineering as well. In these fields it is very common to have modal analysis of structures with many eigenvectors and it is necessary to select only the most important modes. These modes are the target modes.

The modal analysis of a moving assembly supplies several axisymmetric modes, and only some of them are indeed relevant for the final acoustic performance of the transducer. See, for example [2], for a general overview of these modes. The problem of the poles needed to develop a compression driver with a wide frequency range was stated by Murray and Durbin in 1980 [3]. These authors established that at the upper frequency range it is needed some pole to have acoustic response at that frequency range. They considered the cases of a pole too low in frequency and a pole too high in frequency as the main cause of a lack in acoustic response at high frequencies. Murray and Durbin deal with the influence of the suspension on the high frequency pole. This article has the intention to continue the analysis started by these authors adding more points of view to this problem. For simplicity all effects dues to the diaphragm air loading are not deal in the paper. It is obvious that mechanic explanations given in the paper must be extended taking into account these air loads as well. Non basic axisymmetric modes as: the rocking modes, suspension's modes, etc., are important from the performance point of view, but cannot be considered as target modes defining the acoustic frequency response function.

2. SHORT FORM ELEMENTARY PHYSICS ANALOGY AND MODES, FOR AN EXTENDED BANDWIDTH TRANSDUCER

For sake of simplicity the dynamics of a moving assembly of a compression driver can be better understood by means of the analysis of a single slender beam suspended by two thinner beams at the ends, as Figure 1 depicts.

Professor J. P. Den Hartog [4] explained the dynamics of a beam in bending, and he said: "The physical characteristic of any normal elastic curve of the beam is that the q (load per running inch) loading diagram must have the same shape as the deflection diagram". "Any loading that can produce a deflection curve similar to the loading curve can be regarded as an inertia loading during a vibration". This was stated in a similar manner by Timoshenko and Young [5]. Notice this asserts are the basis of the Rayleigh method to find the lowest eigenvalue of a structure or a system, which has been for long time used solving engineering problems.

Figure 1: Thin round bar supported (suspended) by two short very thin round bars at the ends.

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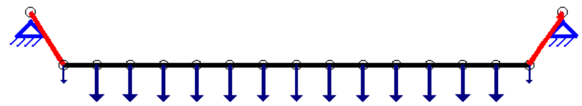


Figure 2: Static response of Figure 1 (beam and suspension).

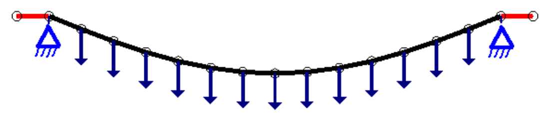


Figure 3: Static response of the beam alone of Figure 1.

Assume we have a slender beam which has a very narrow short beam at their ends. The beam will take the function of a moving assembly, and the short and thin beams will take the function of a suspension. This elementary device is depicted on Figure 1. If we load statically the full beam of the Figure 1 we will find, as expected, a deflection curve with the shape of the Figure 2. This response is the one found in moving assemblies of loudspeakers due to the huge bending stiffness difference of the moving assembly and suspension. The usual well known static response of the beam without any suspension, loaded by its own weight, must be found applying the displacement restrains to the beam's boundary. This response is depicted in Figure 3.

However, when the suspended beam is submitted to dynamic loading or vibrations the first mode shape is the one depicted in Figure 4. If the beam is restrained by its ends and not by its suspension's bars and if it is submitted to the same dynamic loading, the first mode shape is the one illustrated in Figure 5. Notice that these mode-shapes or eigenvectors have the same shape as their corresponding static loading depicted on previous figures. This is, in short form, the statement of Professor Den Hartog [4].

If the body stiffness is much higher than the suspension's stiffness, the eigenvalue of the mode-shape depicted in Figure 5 is much higher that the eigenvalue corresponding to the modeshape of Figure 4. This is well known by practice and by common sense. Both modes depicted on figures are the target modes for this slender body.

Strings and bars have been treated for long in mechanics books.

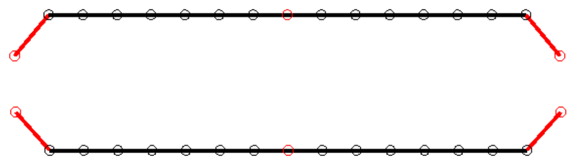


Figure 4: Dynamic response. First mode-shape of the beam –suspension set of Figure 1.

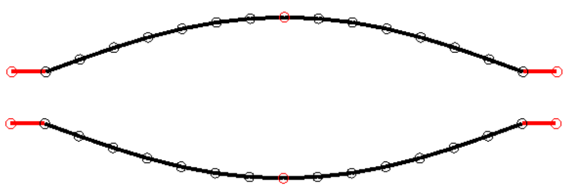


Figure 5: Dynamic response. First mode of the beam of Figure 1 when beam it is restrained by its ends but not by the beam's suspension.

2. 1. The mode shapes in moving assemblies of compression drivers.

In compression drivers this simple correspondence between static and dynamic load can be applied as well. The main natural frequency and its corresponding mode shape of a compression driver of the dome type are generally well known; but the second target resonance is generally less well known. Figure 6 depicts a dome compression driver and the downward of the mode shape when the transducer is performing the well known low frequency resonance. The Figure 6 may be obtained just by static loading of the moving assembly by its own weight.

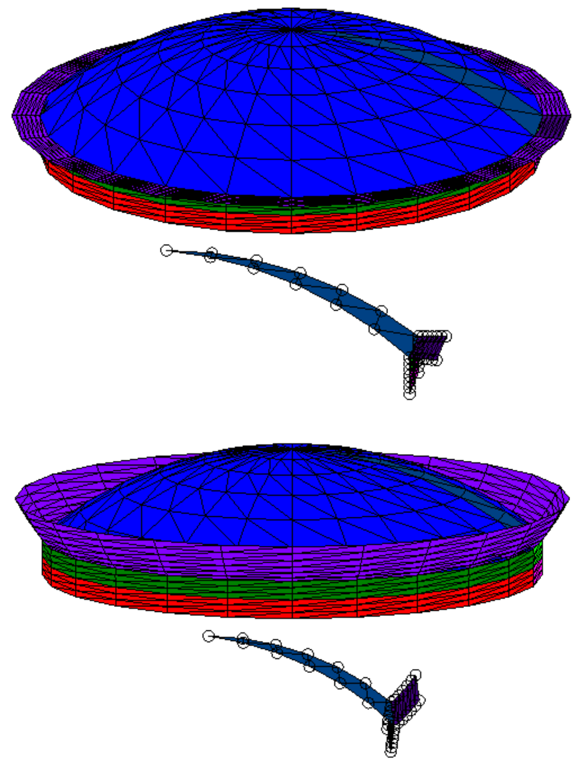


Figure 6: Upper: Coil, Former and Dome of a compression driver with its suspension. Bottom: Shape of the moving assembly after loaded statically by its own weight.

However, the second mode referred by Murray and Durbin, which in fact it is an important or a target mode, is much less known and often difficult to find by the design staff. Modal analysis of moving assemblies of compression drivers gives several eigenvalues and mode shapes which must be ranked in importance for the right goal of finding acoustic output at high frequencies as explained in [3].

If we restrain the outer dome's nodes instead of the outer suspensions nodes, and then we load the dome by its own weight, we will obtain the shape depicted in Figure 7. The restrained nodes are highlighted on the Figure. We will see later that this deformed body can be found as a mode when calculating the modal analysis of the body. In the case a single mode do not has the same shape, this shape can be found as a linear combination of some specific modes. This mode shape is the one focused by Murray and Durbin on their paper [3].

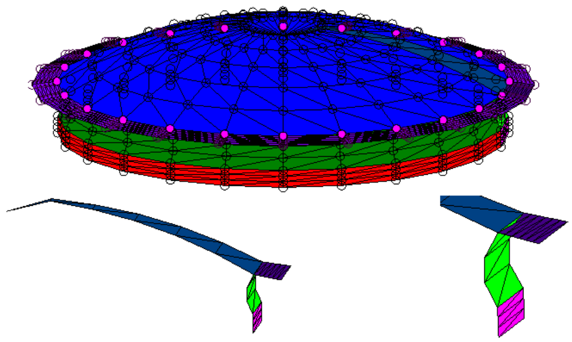


Figure 7: The moving assembly after loaded by its own weight. The body is restrained in translation for all degrees of freedom at the dome-former interface (these nodes are highlighted). Observe details of the deformed former and the dome's pole in the inset.

3. GENERALIZED MASSES, PARTICIPATION FACTORS, AND STRAIN ENERGY OF A SLENDER BODY

The main tool to find important and target modes is the value of the generalized mass. The definition of this concept can be found in books as [6] and papers as the ones in [1], [7] and [8].

This concept, and the participation factor, has been widely used in the field of seismic engineering. This is because, as in space engineering, the target modes must be enhanced respect to the many found in modal analysis of large bodies.

Other aspects of interest for the use of the generalized mass concept are: a) to divide large model systems in subsystems [1], and b) separation of modes which have a certain overlap in spectrum [7].

If we have a system defined by:

$$\mathrm{M}\ddot{x} + Kx = F \qquad (1)$$

Being: M: the mass matrix, K: the stiffness matrix, F: the forcing function, and being x and $\ddot{x}$ the displacement and acceleration vector respectively.

The solution of the system (1) is found in terms of eigenvalues and eigenvectors. Being, as was said before, the eigenvectors the vibration mode shapes. Let Φ be the eigenvector matrix.

The system's generalized mass matrix $\hat{m}$ is given by:

$$\hat{m} = \Phi^T M \Phi \qquad (2)$$

Generally, those modes which have high generalized mass are modes that are highly excited and they have relevant importance in the system's response. This will be shown in the following paragraphs. See Appendix A for further details about the physical meaning of the generalized mass.

The generalized mass concept is very important in dynamics because can be associated to the coherence concept in acoustics and vibrations. Notice that the generalized mass will be maximal when the mass matrix will "fit" with the mode shape, see equation (2). Notice in this equation the mode shape is represented by Φ and by Φ^T as well. Observe that in dynamics, we may have a motion of a certain point of a system in a specific direction because the action of inertia force(s) acting in other part(s) of the body and not necessarily in the same direction. The coherence is high if the motion of each part of a system is due to inertia force acting on the deformed point and applied in the deformation direction and sense. Simply speaking, we can say the coherence is high when the responsible of the deformation of a system's part is the inertia force of each own deformed part. This is the cause of the high coherence of a pleated tweeter. In these transducers the acting forces deform the pleated moving assembly at the same points of deformation and at the direction of the needed mode shape. Thus, in these transducers the generalized mass is high for many modes. A high generalized mass for a specific mode which has a mode shape or eigenvector Φ_n must have the amplitudes of all nodes as high as possible in order to increase the products of equation (2). For example, in the main mode of a speaker all node masses of the moving assembly move with the same amplitude which equals 1. In this case, for this specific mode, the generalized mass, equals the moving assembly mass. This is the cause of the high excitation of the main mode in a speaker. This asserts might seams obvious, but as we will see, there are other interesting aspects of this topic.

The participation factor is another important parameter that has full application in the electro-acoustic transducers field. In loudspeakers it is very important to target the modes which excite masses only in the excitation direction, and the participation factor takes this it into account. Translational components of the participation factors are defined as:

$$r_{iv} = \frac{\sum_{j=1}^{N} \Phi_{ij} M_{ij}}{\{\Phi\}_v (M) \{\Phi\}_v^T} \quad (3)$$

Where:

Symbols are the previously defined and the rest are as follows:

i: is the component identification (the degree of freedom) (1 to 6)

j: is the node identification (1 to N nodes the model has)

v: is the mode identification (1 to V modes). V is the total number of modes found by the model extractor

N: is the total number of modes.

Thus, the participation factor r_{iv} for the mode v at the i direction equals the summation of all displacements j of the eigenvector Φ, times the associated masses divided (normalized) by the generalized mass of this specific mode. Observe the participation factor is an interesting concept because takes into account the global mass associated to one specific direction, and the coherence of this mass in motion due to the phase or the motion sense of all masses. Notice that at high frequencies (where the second target mode is activated) modes are localized, which implies loss of generalized mass. Thus, as the participation factor is normalized to the generalized mass, any localized mode will have a loss of the participation factor as well.

A mode will be target if the generalized mass is high, and it is convenient that the axial participation factor shall be high as well. With this concept in mind it is easy to find which modes are target when designing a moving assembly

The strain energy is equivalent to the generalized mass in the sense that the mass is the load of the system and associates the kinetic energy of it. Nevertheless the strain energy represents the elastic energy the body has and that it can be transferred to the surrounding air. This elastic energy is the available potential energy the body has to compress and expand the neighbor air.

For definitions of the strain energy see for example [9] and [10]. It is basic to write down the definition of this concept. For shortening it will be referred to beams but the extension of this concept is similar for shells.

Total potential energy or the elastic energy Π of a beam loaded axially by the load P is:

$$\Pi = \frac{EI}{2} \int_0^L \left(\frac{1}{\rho}\right)^2 dx + P \int_0^L \frac{du}{dx} dx \quad (4)$$

Being: $\frac{1}{\rho} = \frac{d^2u}{dx^2}$, u(x) and w(x) are axial and transverse displacements measured from the unreformed axis of the beam. E and I are the Young modulus of the beam and the inertia moment of the beam cross section respectively. L is the beam length.

Notice the first term of the right side of (4) is the bending energy, and the second term is the in-plane energy strain of the deformed beam.

Figure 8 depicts the energy strain of the deformed moving assembly of Figure 7. Observe the elastic energy the body has is not necessarily were the deformation is greater (the dome center) but where the potential work is higher, which is on the dome interface with the former. This is in conformity with the explained in [11]. This point will be seen in depth later on.

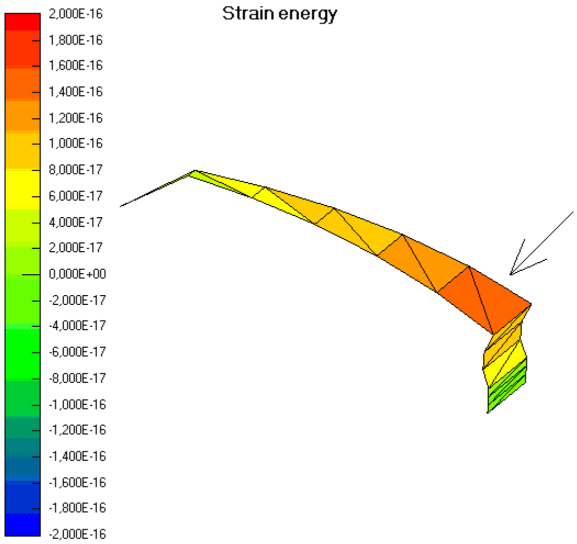


Figure 8: Strain Energy diagram for the deformed moving assembly of Figure 7. Arrow shows the element with highest strain energy

3. 1. Stability of structures. The minimum potential energy criteria.

The stability of structures has been an important topic of physics since the beginning of the science studies. Lagrange's theorem [10], [22] establishes that if the potential energy of a structure has a local or isolated minimum in the equilibrium position, then, the equilibrium is stable.

From a practical point of view [10], today it is accepted that: the existence of a (weak) proper minimum of the potential energy in the equilibrium state constitutes, for all practical purposes, both a necessary and sufficient condition for the stability of this configuration in the sense of Lyapunov. It is reported [10] that in some situations, and for some structures, the state of zero strain and stress may be unstable as well.

3. 2. Some instabilities of moving assemblies. Target modes which its potential energy is higher that the neighbor modes.

In the case of dynamics of moving assemblies the problem can be stated in some different manner. While a suspended moving assembly is subjected to vibrate in a certain frequency range, it will exhibit modes. Each mode has its specific potential energy. Thus the modes which its potential energies are maximum in respect to its neighbours are less stable or even unstable. This can be the case of sub harmonics generation in some transducers, see [21] for example. If a transducer is forced to vibrate in a mode which is not stable, one way to escape of this mode is to migrate to a lower mode or to vibrate in a sub harmonic frequency. There are several practical performances on this situation, being one very common in practice to vibrate in two modes simultaneously, etc. Observe the sub harmonic is produced because the transducer is forced to vibrate in a mode which elastic energy is high. Sub harmonics need a certain voltage level to appear. If we increase the applied voltage at a frequency which corresponds to a mode shape of high elastic energy, the excess of energy supplied to the moving assembly is dissipated performing longer cycles, and one of these forms is the sub harmonic generation.

4. TARGET MODES IN COMPRESSION DRIVERS.

It is seldom in practice to have a single geometrical body as a single sphere or a single cylinder. Despite it is convenient to have moving assemblies as simple as possible, generally, they must have a coil, a diaphragm and a suspension.

4. 1. Moving assembly of two bodies which curvatures are in opposite directions. The axial mode.

Imagine the theoretical case we would have a body which is formed by two hemispheres jointed by their poles, as it is depicted in Figure 9. This theoretical device it is suspended isotropically (by 6 springs) by the joint point of the hemispheres, which is the symmetry center of this body. Observe that this slender body has two curvature centers that are at the opposite sides of the suspension point. The total weight of this set of two stiff and thin hemispheres is 14.9grams. The radius of each hemisphere is 76.76 mm. Due to its geometry (stiffness distribution) and to the mass distribution, this body beside the main mode, trends to vibrate in a mode which is axial. This axial mode is one of the most relevant for this particular theoretical device. Figure 10 depicts this theoretical body mesh at rest. Figure 11 depict the two hemispheres performing both the main target mode and the axial target mode as well. The generalized mass for the main mode is 14.9 grams; and for the axial mode it is 14.04 grams, which is very close to the total body's mass. Obviously, the main mode has reached the total model mass but the axial mode which happens at high frequency has a high generalized mass as well. Generalized mass reflects the amount of mass involved on a specific mode. Apendix A gives deeper insight on this concept.

However things are totally different from the participation factor point of view. The main mode has an axial participation factor of one, while the axial mode has a value close to zero. This is because the counter-phase response of one hemisphere respect to the other in the second mode. From the strain energy point of view the global elastic energy of the full body for this axial mode is fairly low. This is because the nature of the elastic energy is of the bending type.

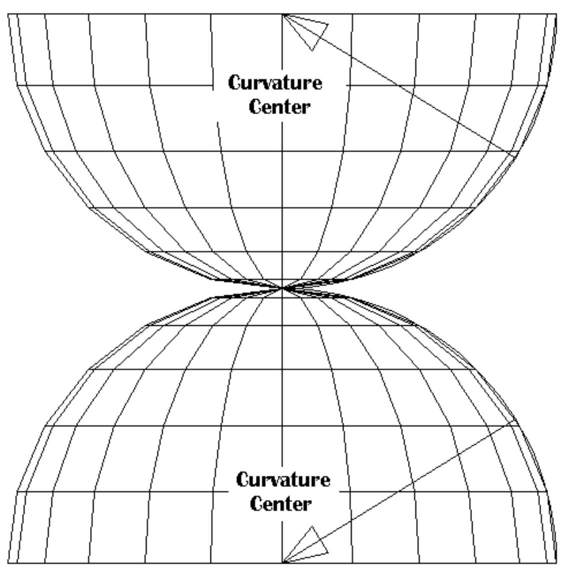


Figure 9: Two hemispheres jointed by its poles. This is an example of a compound body, with curvatures in opposition. The body is suspended isotropically by the common node.

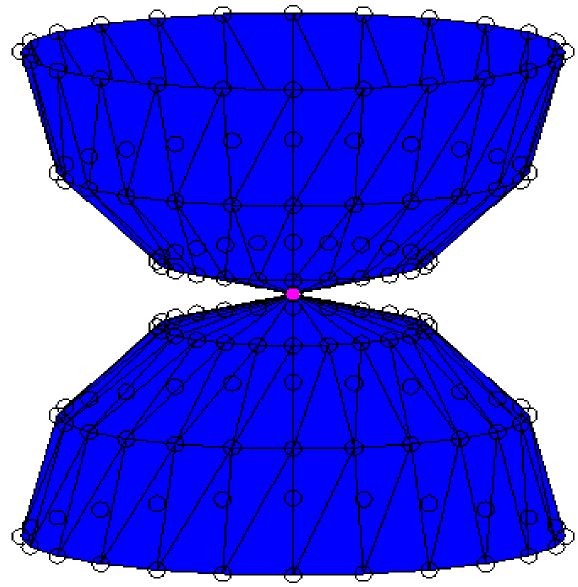
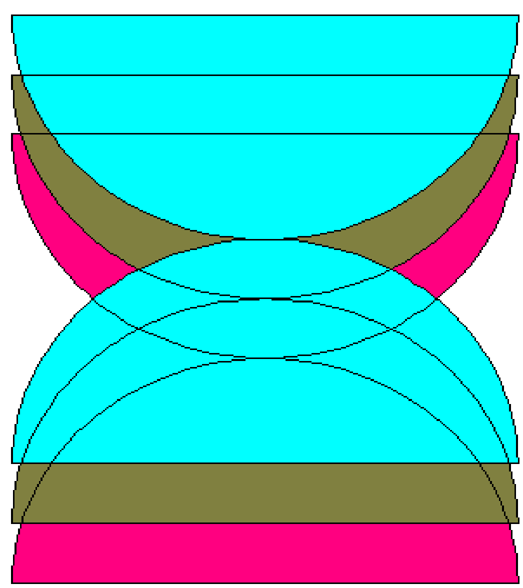


Figure 10: Finite element mesh of the theoretical device of Figure 9. Body is suspended isotropically by the interface node. Model is at rest.



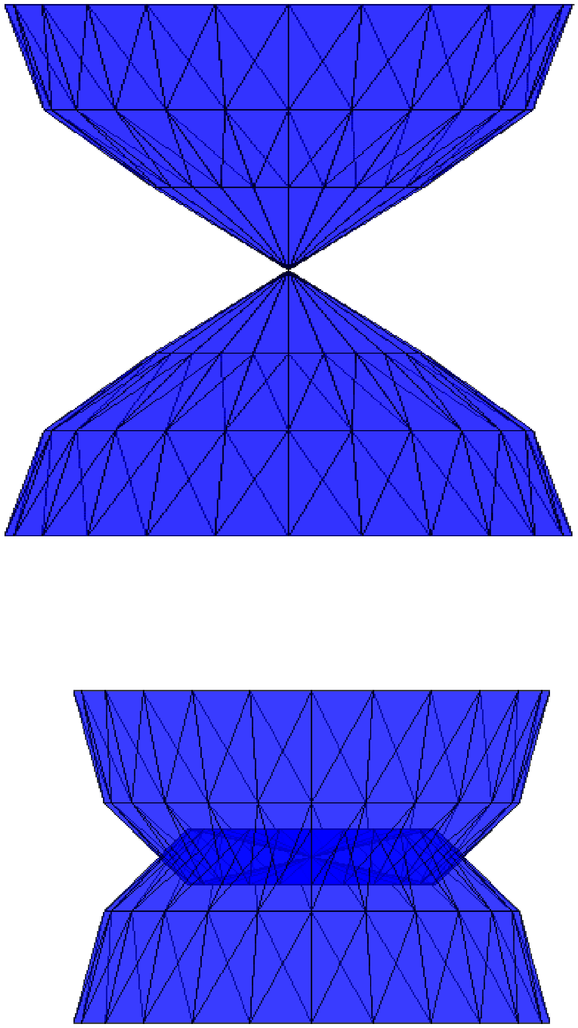


Figure 11: Top: Main mode of the compound body of Figure 12. Middle and Bottom: Axial mode of this body. The generalized mass calculated for this mode is 14.04grams.

4. 2. Moving assembly of two bodies which curvatures are in the same direction: the breathing mode.

If we joint the two hemispheres by their maximum circle, we will have a full sphere. This body will be suspended orthotropically by the equator circle. For this theoretical case we have the two curvature centers for the two hemispheres in the same point in space which is the sphere center. Modal analysis of this body has an important mode which is the sphere breath. The sphere open and closes synchronized by the in-plane forces. While the sphere exhibits this particular mode the generalized mass has the value of 14.37 grams. Observe the calculated generalized mass is almost the full sphere mass. This indicates the importance of this specific mode. However the participation factor must be defined in one specific direction in Cartesian coordinates. As the sphere breaths radial, and not axially, the participation factor for any Cartesian coordinate is almost null. Acoustically the sphere delivers coherent sound waves when it is breathing. But, unfortunately the participation factor is not useful for a breathing sphere. The strain energy of the full sphere is large for this specific mode. The elastic energy is also large, because the nature of this energy is of the in-plane type. Notice this mode can not be obtained by any dead weight loading of the body (by static analysis). To obtain that specific mode the inertia forces must act in all space directions. The breath mode, as the reader can see, is indeed a basic mode.

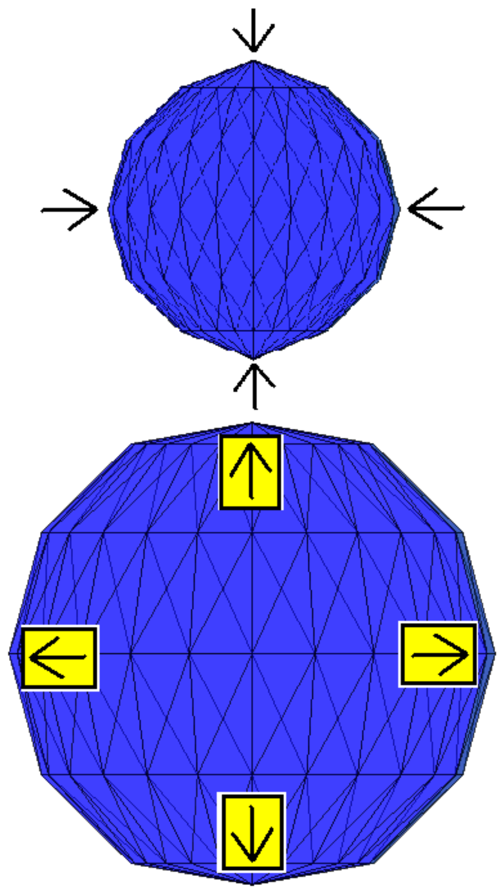


Figure 12: Sphere orthotropically suspended by its equator, performing the breath mode. Generalized mass calculated is 14.37 grams.

4. 3. The breathing mode in cylinders.

This mode consists on a general extension and contraction of all cylinder particles. This mode is generally known as ring mode, see for example the references [12], [13], [14], [15] and the work of [16], which deals about it in depth.. This mode is very important because all acting forces are in-plane forces being full coherent. The mode is depicted in Figure 13. In the figure we see the former and coil's breath. Each individual breath is depicted on the Figure.

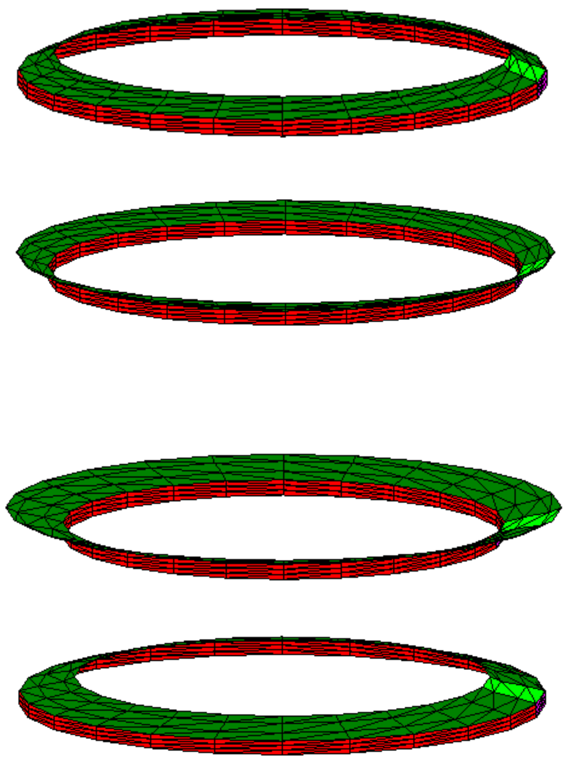


Figure 13: Upper Breath mode of the coil's former. Lower: Breath mode of the coil.

4. 4. Experimental measurement of the breath mode in a voice coil.

Figure 14 depicts a simple experiment of ring frequency excitation in a coil. The aluminum voice coil has 100 mm diameter and 3 mm height. Voice coil is submitted to an axial magnetic flux as figure shows, while a sine sweep voltage is applied to its terminals. Due to the voice coil aspect ratio, the radial resultant force will excite the ring mode much hardly than in the speaker configuration, because the ring mode axial component is much lower than the ring mode radial component. Figure 15 depicts the acoustic radiation of this coil for a flat white noise spectrum voltage applied to its terminals. The measurement where made in the near field and the gap between coil and microphones was kept as equal in both channels as possible. Observe the sharp peak at 13088 Hz which corresponds to this ring or breathing mode. The former was removed leaving the coil free and the measurement was repeated again. The breath frequency rises to 14200 Hz because the former only add mass to the dynamics but not stiffness to the aluminum wire.

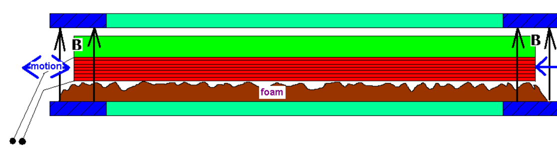


Figure 14: Coil submitted to an axial magnetic field while intensity is circulating. Forces are radial and the body is vibrating "breathing". The body below the voice coil it is soft foam.

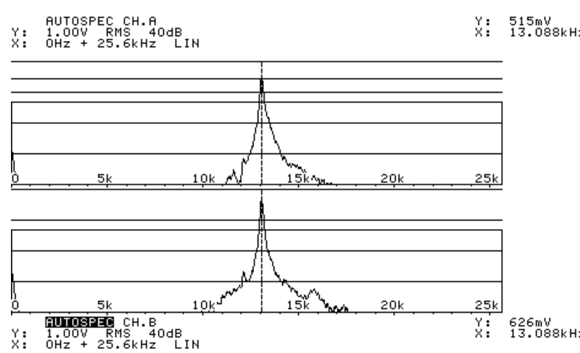


Figure 15: Radial acoustic response on the near field of an aluminum coil with its former. Coil's diameter is 100 mm, and the coil's height is 5 mm. Coil's breath sound was measured by two microphones arranged radial 90 degrees apart.

4. 5. Breathing mode in compound bodies.

The moving assembly is the joint of two symmetric bodies: a spherical dome and a cylinder. Figure 16 shows the moving assembly. This combined body has a global breath mode as well. Due to the mismatch of the meridian and tangential stress [11] at the former-dome interface, high shear forces will appear at this contour. Due to the high stiffness of the dome, the coil-dome's breath compound mode has a natural frequency which is higher than the one seen on the last paragraph. Despite the moving assembly breath mode exists as well, it is not a pure breath mode consisting of high coherence. The presence of shear forces at the interface decreases the coherence of that mode. However this can be considered a compound breath mode.

The moving assembly performing the compound breath mode it is depicted in Figure 17. Observe the opening and closing of the coil's and the extension and contraction of the dome contour. The complexity of this compound breath mode add bending at the dome's pole, but the elastic energy of this bending is very small. Notice the dome is very shallow.

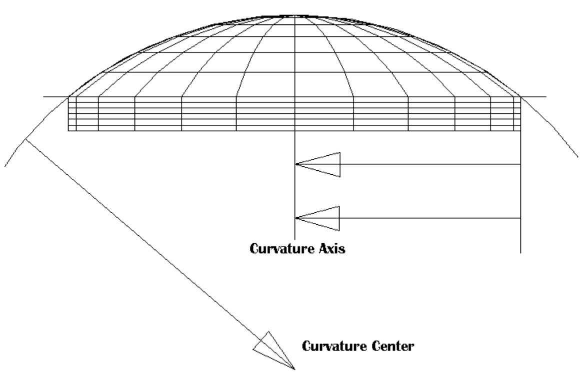


Figure 16: Spherical segment and cylinder which curvatures are at the same side.

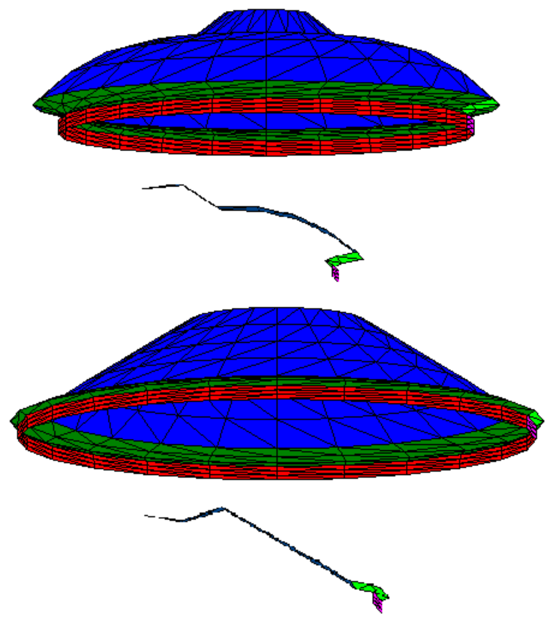


Figure 17: Spherical segment and cylinder performing a compound breathing mode. Observe the coil's ring motion.

4. 6. Main axial mode in compound real bodies.

Due to the fact that the dome's mass center it is above the interface contour line, and that the coil's mass center it is below the same contour line, this body, which is structurally weak at the former, has a trend to exhibit a main axial mode as well. This mode it is depicted in Figure 18. Observe in the figure that the coil does not breath at all. The dome extends axially. Due to this, the dome's pole bends as shown for the compound breath mode. Despite dome's pole is bending with high amplitude, the strain energy it is small, because do not contribute significantly to the global strain energy of the moving assembly. It is convenient to analyze results in terms of response amplitudes, stresses and energies, but not by "pictures". Notice that the spherical dome when loaded axially, it is soft in bending, at its pole area. At this axial mode the former it is highly stretched.

Following the idea of 4.1 paragraphs, when the curvature centers are at bodies opposite sides, then the target axial mode it is highly activated. Figure 19 depicts one of these designs in electro-acoustics. This moving assembly has the main masses at opposite sides and has bending capability between both masses. Tuning the split masses and the bending stiffness of the lower spherical segment, the axial target mode can be activated. This moving assembly trends to exhibit an axial mode, but due to its geometry, do not trend to exhibit a compound breath mode.

This axial mode is depicted in Figure 20. The bending mechanism at the lower spherical segment it is clearly visible in the figure.

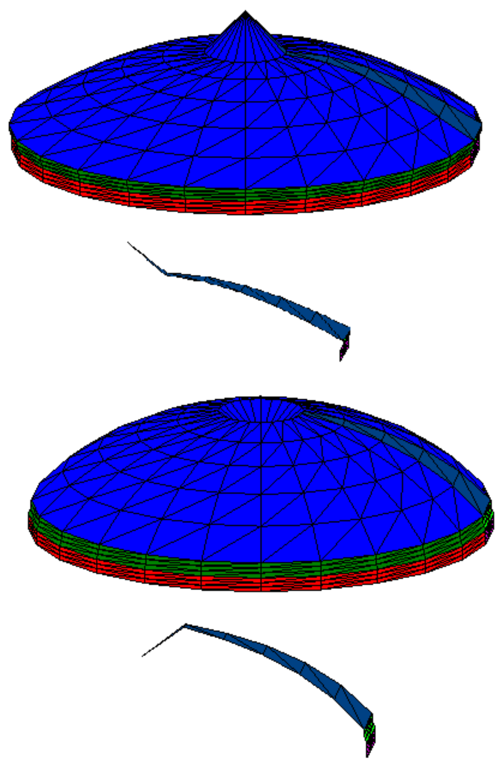


Figure 18: Spherical segment and cylinder performing an axial mode. Dome is bending but the coil's former it is bending and stretching as well. See the explanations of pole bending on text.

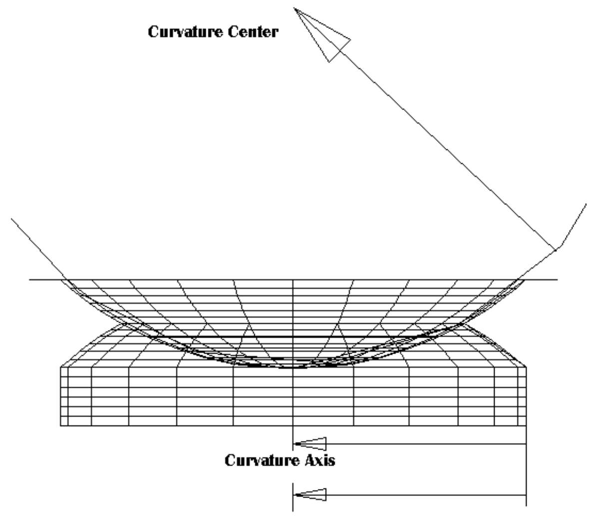


Figure 19: Moving assembly of two bodies which curvatures are at opposite sides.

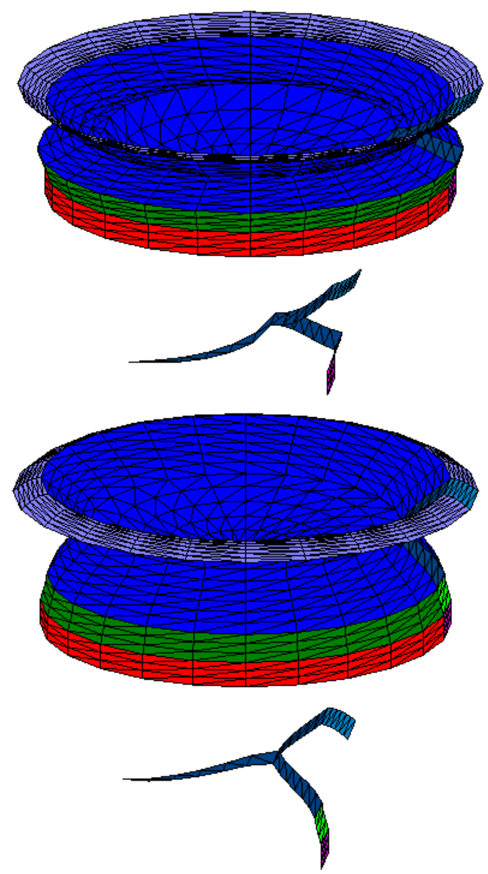


Figure 20: Main axial mode of the moving assembly of Figure 19.

5. COMPETING MODES

5. 1. About the zeroes of a driving point mobility function.

Murray and Siefert [17] made excellent motional impedance measurements. Some results are extreme difficult to obtain, especially those referred to the zeroes; this is mainly because the zeroes are points close to the measurement noise. The authors measured the moving assemblies at free air and in vacuum as well, and they used a dummy blocked transducer for discounting the pure electric impedance over the global transducer's impedance. The subtraction of the electric impedance of the global impedance it is very useful, providing graphs of simple interpretation. Some poles found by the authors were not able to sustain the frequency response at high frequencies, where these poles were needed. Some poles which do not sustain the desired acoustic response they are placed very close to a nearby zero in the spectrum. These very close zeros, in some extend, neutralized the effect of the desired poles. Unfortunately, the authors do not gave explanation of the zeroes of the functions they measure. As explained, they put their focus only on the poles for an extended frequency range of the transducer.

When testing the mobility of a structure we relate the force, which generally is the cause of motion, with the velocity which is the effect of this cause. On mechanical measurements there are two important functions depending on which structural points the velocity and the force is measured. If these points are physical apart, the measurement is called transfer mobility. If velocity is measured at the same point that the force it is applied; then the measurement, it is called driving point mobility. When measuring the impedance of a transducer, we relate the voltage at the voice coil terminals, which is proportional to the coil's velocity, with respect to the intensity, which is proportional to the available force on the transducer. Both electric parameters are measured at the voice coil, which can be treated as the driving mass of the moving assembly.

In literature is seldom treated the subject of giving physical interpretation of zeros, but this physical meaning simplifies the understanding of some laboratory measurements and specifically the impedance curves.

The first point of interest is to relate the zero spectral position, in respect to the adjacent poles, related to the masses involved on the dynamics. Figure 21 depicts the velocity spectral function of the coil and the associated cone of an ideal speaker with a total moving assembly mass of 80 grams. This speaker has the cone neck able to deform axially, allowing the voice coil and the cone being dynamically split in two masses. Black curve represents the velocity of the coil for a heavy voice coil of 70 grams, while red curve is the velocity response of the coil for a light voice coil, of only 10 grams. Observe the level difference for the two cases and the spectral position of the two poles, which is obviously, the same. Observe how the zero is closer to the second pole for the heavy voice coil, and observe how the zero is closer to the first pole for the light voice coil. The acoustic responses of these two theoretical loudspeakers will be completely different, not only because the global level is much less for the speaker of a voice coil of 70 grams, but also because the zero of the speaker which coil weights 70 grams is very close in the spectrum to the second pole.

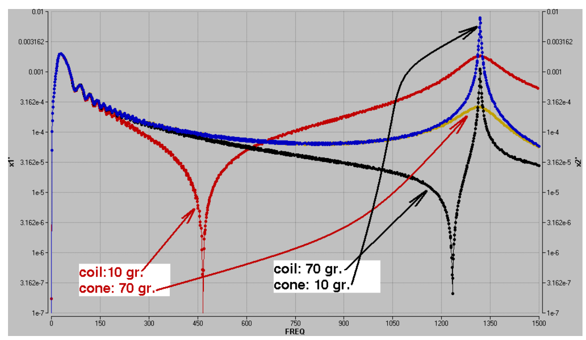


Figure 21: Velocity spectra of a light (10 grams) and heavy (70 grams) voice coil attached to its associated cone. The moving assembly has a fixed total mass of 80 grams. Velocities of the associated cones are depicted as well.

5. 2. Driving point mobility measurements on two masses linked by a spring.

Because the subject's interest, experimental measurements done in a two degree of freedom system are presented here. The experimental set is depicted in Figure 22. A hammer with a force transducer impacts the tip of each mass on axis, while an accelerometer pick up the acceleration signal at the same excited mass on axis as well.

Figure 23 depicts experimental modulus of the driving point mobility function of two masses linked by a spring. The masses are very different in weight, and the system's driving point mobility was tested at both masses. Upper figure is the driving point mobility of the small mass, while bottom figure is the driving point mobility of the heavy mass. Observe the full scale value of the upper figure is 200 m while the full scale value of the lower figure is only 50 m. Observe the position of the poles and the zeroes. First pole is beyond left side of figure because the masses were suspended by a string and the first natural frequency it is very low, beyond the graph. Notice the zero it is difficult to be measured because it is buried into the measurement noise. But despite this circumstance, the zeroes of the figure follows the spectral position same criteria than the ones obtained theoretically on Figure 21. When the small mass it is excited by the force transducer, the zero between poles appears far apart of the second pole, which is the desired pole of the Murray's and Durbing's work [3]. Whereas the heavy mass is excited with the force transducer, the zero root appears close to the second pole.

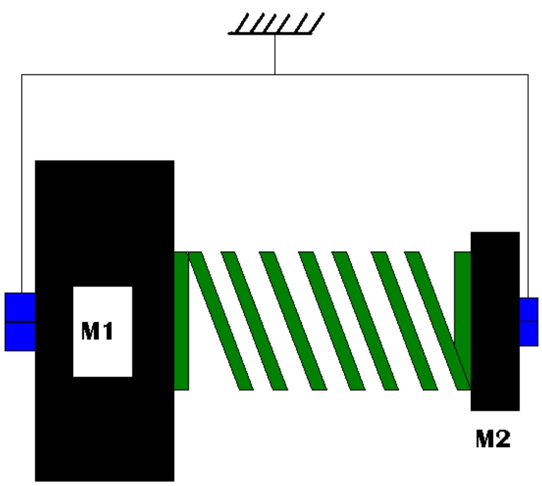


Figure 22: Experimental two degrees of freedom system for driving point mobility measurements.

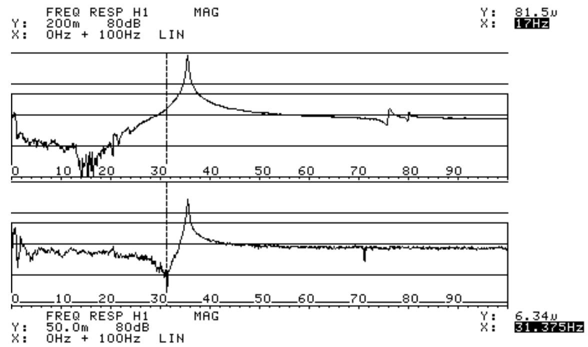


Figure 23: Driving point mobilities of the two degrees of freedom system of Figure 22. Upper: mobility of the light mass. Lower: mobility of the heavy mass.

The explanation of the zero position between poles is cumbersome and not fully given [18], [19], but a rough idea can be given here. Reference [20] treats the problem of the zeros splitting a system in subsystems. Splitting the system in subsystems gives a much better point of view of the physical phenomena. Zeros are characteristics for those input signals that can be totally blocked by the system dynamics.

The reader will remember that the mode-shapes of the masses of Figure 22 are in phase for the first mode and in anti-phase for the second mode.

As in the first mode the spring between the masses do not absorb any potential energy (because it is not deforming) both masses act as a single one (of a global weight) absorbing the external input energy.

However, the situation is completely different for the second mode where the spring deforms and absorbs external energy. When the force is applied to the big mass of Figure 22, most energy is absorbed by the input mass, this energy is kinetic only. This kinetic energy is the source that acts on the spring to deform it. Because the external force it is applied to this big mass, the energy attacking the spring is the delivered by the external force loaded by high internal impedance, which is the big mass. Thus the energy available for the second small mass to perform the second mode it is clearly reduced because the input energy was applied to the big mass. The lack of energy available for the second mode is the "band stop" or the blocking action the system has. Nevertheless it is obvious that if the input energy were applied to the small mass, the amount of energy available for the second mass to perform the second mode is much grater than the contrary case. In terms of a sweeping frequency the problem can be seen as follows: First the system is tuned to the first mode and both masses move synchronized in phase. Once the first mode has been past in frequency, the two masses must change their dynamics to a motion which is in anti-phase. In order to change the phase from in-phase to anti-phase, the driven mass must stop and change its motion sense until it reaches the anti-phase motion with respect to the un-driven mass. The driven mass stop is the system band stop or energy block of the input subsystem. It is obvious that, due to the nature of inertia forces, while sweeping the excitation frequency from low to high frequency, it is easier to stop before a small mass than a big mass while sweeping. This is the cause of a low frequency zero when the voice coil is light, and it is the cause as well of a high frequency zero when voice coil is heavy.

5. 3. Experimental evidence of competing modes. The suspension's effect over the moving assembly.

Figure 24 depicts the in vacuum motional impedance function of a large diameter (100 mm diameter) dome compression driver. Scale is in arbitrary units but calibrated on dB; full span is 40 dB. Observe the zero at the cursor frequency (15104 Hz). Observe the small cluster of small poles marked with arrows at the right of the zero. The main high frequency pole is split on various peaks. This peak split can be caused by a dome that is not able to keep the structural stability. Observe the small peaks left to the zero (one is marked with an arrow). Peaks like these can be what Murray and Durbin called "activity" [3].

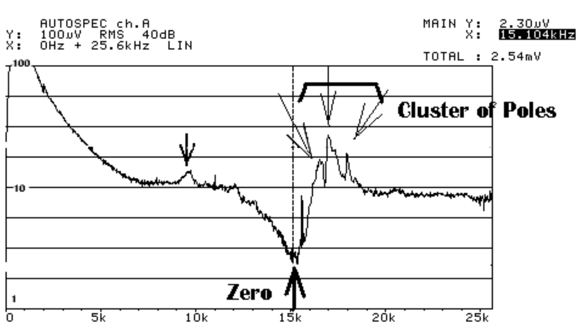


Figure 24: In vacuo motional impedance measurements of a dome compression driver of large voice coil.

After 5 hours that this transducer was in vacuum, it was submitted to the same measurement. The result is depicted in Figure 25. Notice the poles are the same but the zero has shifted to a very low value (8 KHz). This performance is due to the fact that the bulk of masses of the moving assembly changed their distribution after staying the moving assembly 5 hours in-vacuum. The zero's spectral shift implies a non linear behavior of the moving assembly and suspension set. This nonlinearity is totally consistent with the idea of a dome which is not structurally stable [23].

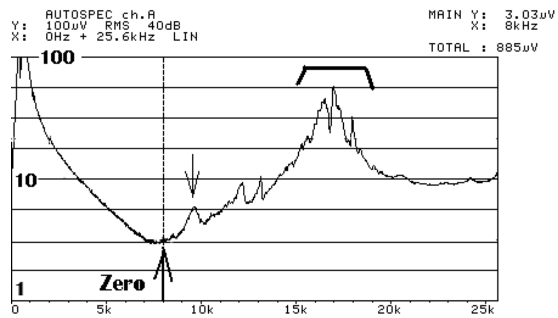


Figure 25: In vacuo motional impedance measurements of a dome compression driver of large voice coil after 5 hours in the vacuum chamber.

Murray and Durbin emphasized in their paper that the moving assembly's suspension change was able to modify drastically the acoustic response of the transducer. But unfortunately the authors do not give full explanation of the suspension's action on the moving assembly response.

The most plausible explanation is that the moving assembly has two basic high frequency modes: the compound breath mode and an axial mode as we have seen before. Dome transducers of big size have both modes in the audio frequency range, and they are not too far one from the other in spectrum. Small size dome compression drivers have these main high frequency modes beyond the audio frequency range, normally. These basic modes can compete in some circumstances. As we saw before for both modes, the coil's former deforms substantially and, in some cases, it is not easy, at the first glance, to distinguish between them. The mode competition may be clearly solved (one mode overcome the other) by the radial forces of the suspension. These radial forces acts on the dome contour, where the strain energy is very high, and these suspension radial forces are close to the former contour. Thus the suspension is able to influence which mode will be activated. This hypothesis is consistent with the cause of sub-harmonic response of these transducers because the action of periodic forces on the dome-former-suspension interface contour, see for example [11] and [21]. Figure 26 depicts a complete moving assembly with its undulated suspension and the effect of the radial forces acting on the moving assembly. Observe how much this shape resembles in some sense the breath mode and the axial mode as well. Observe how these suspension's radial forces split the full moving assembly in two subsystems (dome and former-voice coil), and the zero spectral position depends basically on the mass distribution of the two subsystems. Radial forces illustrated on Figure 26 are depicted independent of the rest of the moving assembly. In practice, generally, the moving assembly mass split will be more complex than the one represented on Figure 26.

Thus, depending on several factors as: the generalized mass of the main high frequency modes, the strain energies of these modes, the influence of the suspension's radial forces, the non linearity, etc, one of these modes can be the one indeed excited. The cluster of poles instead of a single pole is due to the lack of stability of the large diameter and very thin titanium dome. This dome's stability lack is influenced by the industrial conforming process as well.

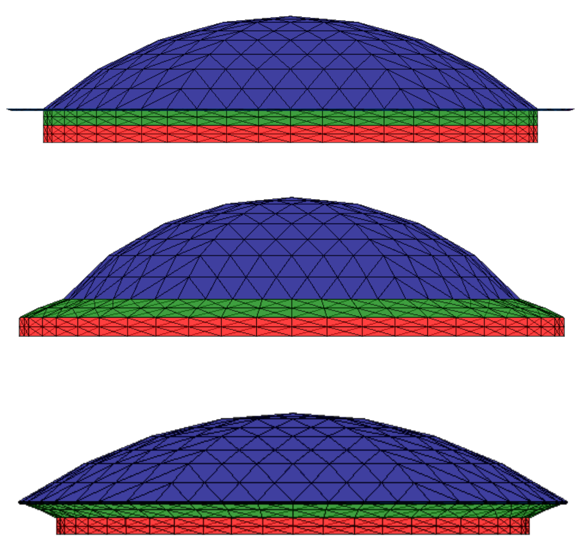


Figure 26: Top: Moving assembly with its suspension. Bottom: Effect of the suspension's radial forces on the moving assembly (suspension is omitted on illustration for clarity).

6. CONCLUSIONS

Target modes are those modes which have practical relevance in a structure. The designers have a difficult task when an extended band transducer is needed. The problem of the second pole for extended band transducers has been treated. The first resonant mode in these transducers is well known, but the second pole is often unknown in practice. Both the axial mode and the compound breathing mode are the active second poles in a dome transducer, while in V shaped transducers, the second pole is the wing's flapping mode (see Appendix B).

Due to its importance, the breathing mode has been treated both numerical and experimentally as well.

The axial mode can be enhanced designing a moving assembly made with bodies which curvature centers are at opposite sides. The dome compression driver can have the axial mode and the compound breath mode competing in some circumstances. The suspension is an active element that may aid to split the global mass in two masses, and can bias the mass sharing. The suspension's influence was mentioned by Murray and Durbin in 1980 [3] but it was not explained in depth.

Analysis of zeros on the motional impedance curve it is seldom treated in literature. In practice, the zeros between poles can act neutralizing the closer active pole. This point has been treated on the paper as well. For large but shallow domes it can be found structural instabilities creating some mode splitting in a cluster. In fact the migration in spectrum of a zero from 15 KHz to 8 KHz it is reported on a practical example.

Generalized mass it is a well known and powerful parameter to establish the main modes. Participation factor gives the main index of mass participation for a specific direction. Due to the breathing nature, the participation factor does not apply well for the extensional mode. When target modes are needed at high frequencies, the strain energy is a useful tool to find them in an often large set of eigenfrequencies.

APPENDIX A: GENERALIZED MASS ON SIMPLE AND COMPLEX STRUCTURES.

Assume we have a mass less bar loaded by an array of equally spaced masses which weights follow the function of the magnitude of a sine function. The beam and the set of masses it is depicted on Figure A1. The beam span corresponds to a complete circumference. Thus for this arrangement the mass step along the beam is $\pi/8$ radians. The bar ends are restrained in all degrees of freedom except the rotation Z (which is perpendicular to the paper surface), and the bar is free to move in X and Y, and free to rotate in Z. The beam it is loaded in X and Y but inertia moments are not loaded. The beam's weight is 1 kg.

Figure A2 depicts the first and second mode-shape of the beam loaded with the masses illustrated on figure A1. The generalized mass as it is defined on equation (2) is:

$$\hat{m} = \Phi^T M \Phi$$

In this case, the elements of the diagonal of matrix M have the same magnitude as the absolute values of the eigenvector Φ and its transpose Φ^T. The calculation of the generalized mass $\hat{m}$ it is trivial.

Then, applying the definition of (2) we have the following results:

Generalized Mass Mode 1	Generalized Mass Mode 2
528 grams	678 grams

Observe that the second mode has a generalized mass which is much closer to the real beam mass of 1 Kg. than the first mode which is only 528 grams.

Notice that the beam loaded with this "exotic" mass array or having a mass distribution closer to the second mode shape will trend to vibrate more in the second mode than in the first one. Observe this does not take into account the mode excitation due to the initial conditions for each mass in a free response test. For this particular example the main target mode is the second mode instead of the first one. Notice, however, the participation factor of this second mode is close to zero; this is because the mass of one half of the beam it is exactly the same as the other half and the motion of two halves is in antiphase. Observe that the low participation factor indicates that this mode is not targeted for a hypothetic transducer, because do not produce a substantial acoustic output (it is a dipole).

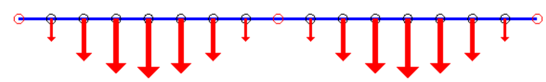


Figure A1: A straight horizontal beam loaded with masses which shape fits with the square of the mode shape of the beam at the second mode.

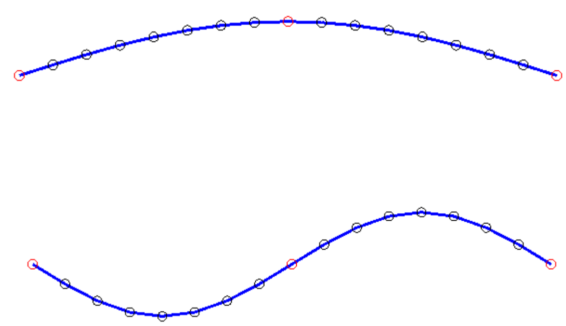


Figure A2: First and second mode of the beam loaded with the masses of Figure A1.

As a comparison the Figure A3 depicts the first and second mode of the same beam free of these exotic loads, weighted by its natural weight, which is 1 Kilo gram and it is suspended by a small flexible suspension.

The analyzed conditions are the same as the previous example, and the generalized masses are:

Generalized Mass Mode 1	Generalized Mass Mode 2
999 grams	253.9 grams

Observe, as expected, the generalized mass of the first mode is almost the global beam mass. For this mode, all values of the Φ and Φ^T vectors equals 1; so that: $\hat{m} = M$.

The second mode has a much lower generalized mass. If this suspended slender beam is vibrated, the first mode will be activated much more than the second mode. The results are close to the ones expected by experience and by the common sense applied to physics.

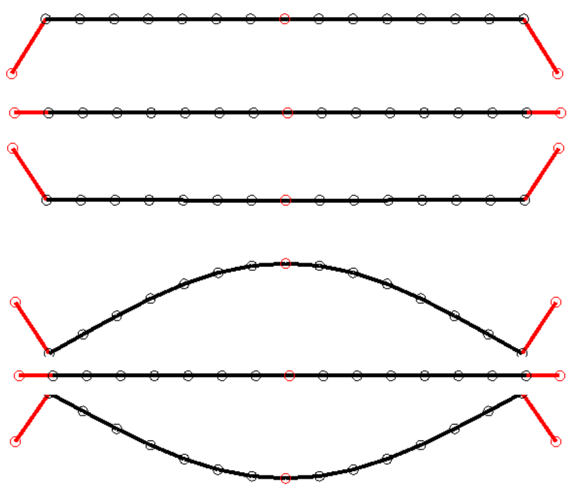


Figure A3: First and second target mode of a round suspended beam loaded by its own weight.

As a complementary example, it is interesting to see the motion of the moving assembly of a pleated tweeter. A moving assembly of this transducer it is depicted in Figure A4. The model was submitted to modal analysis. It is very interesting to realize that the generalized mass of this device it is very high at many modes, which are grouped in clusters by families of modes. This is because almost the total mass of each element of copper, in blue and red colors in the figure, is responsible of the deformation of each element it belongs to. Keeping a very high generalized mass over a wide group of normal modes is a specific characteristic for this pleated transducer. As we have seen in the paper this circumstance is very uncommon in other transducers. The main motion of this transducer is illustrated on Figure A5. Other mode that belongs to the same cluster is the mode 7, which it is depicted on Figure A6; observe the same motion type but grouping the pleats in a different manner. We will not go into details for shortening, but the generalized mass deviation between these individual modes in the cluster it is very small, and the generalized mass is very close to the total moving assembly weight. This is one of the main reasons the pleated transducer provides very good sound.

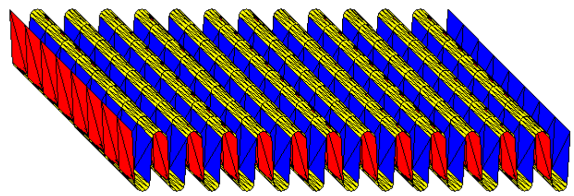


Figure A4: Model of a pleated tweeter.

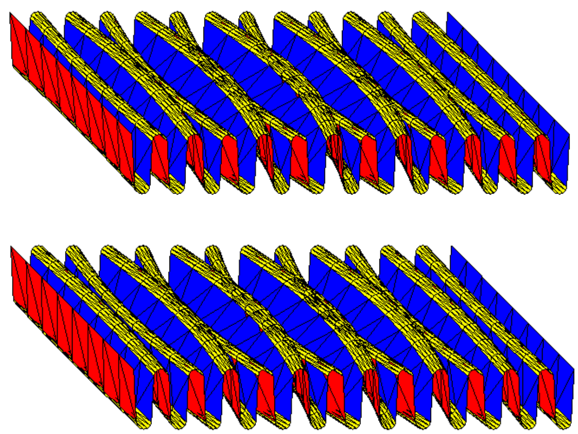
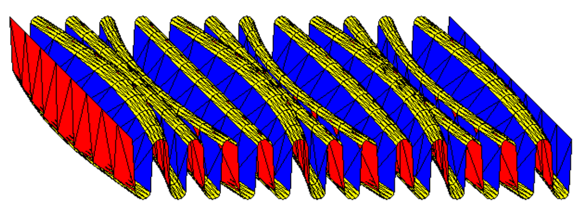


Figure A5: Main motion (mode 2) of the pleated tweeter of Figure A5.



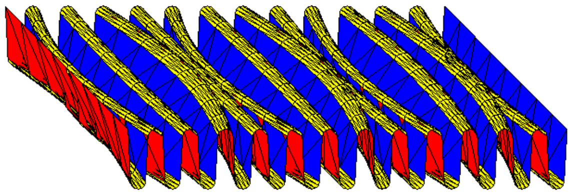


Figure A6: Mode 7 of the pleated tweeter of Figure A5.

APPENDIX B: TARGET MODES IN A V SHAPE TRANSDUCER.

The V shape transducers are widely used in electroacoustics, both in direct radiation and in compression as well. These transducers have an extended response if a strong activation of the second pole is carefully designed.

The second mode can be statically determined following the principles shown in this paper. This is much simpler than performing modal analysis of the moving assembly. Figure B1 shows a V shape moving assembly, provided with a flat suspension. Suspension it is integrated on the V shape diaphragm as a single slender thin shell. Figure B2 shows the two target modes. At the main target mode the diaphragm and coil swing up and down, while at the second target mode the V diaphragm open and closes symmetrically. Notice how in the second mode the wing's cross section of the diaphragm flaps moving the neighbor air or entering air in a compression chamber. This mode, as it must be, it has a high potential energy, and it is the cause of the extended radiation of the transducer in the high frequency region. Observe the similarity with the second target modeshape illustrated by statics on Figure 3 and the one illustrated by dynamics on Figure 5, for a single suspended beam. Observe in Figure B1 the main target mode is less symmetric than the second target mode. The design compromise consists in having a better deformation pattern on the diaphragm, which has revolution's symmetry, for the second target mode, more than for the first one. This is because this transducer is a high frequency unit. The transducer under consideration has a moving assembly mass of 0.43 grams. Notice the generalized mass of the main target mode is only 0.1183 grams. This is because the very small cantilever suspension and the V shape diaphragm design provide a main target mode of poor revolution symmetry. While the axial participation factor of the main target mode is unit. However, the second target mode, which it is very symmetric, has a generalized mass of 0.049 grams and the axial participation factor it is 0.24. Notice this device has the second target mode at 19 KHz. Observe in the figure, the coil it is in antiphase in respect to the diaphragm. Due to the mode localization on the V diaphragm, there is a substantial decrease of the generalized mass. This is the main cause that the axial participation factor will become reduced as well. In any way, for a high frequency target mode at 19 KHz, the generalized mass and the axial participation factors are fairly good in this transducer.

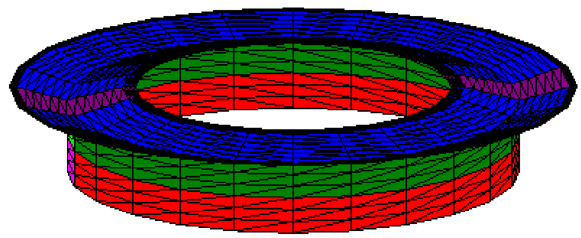
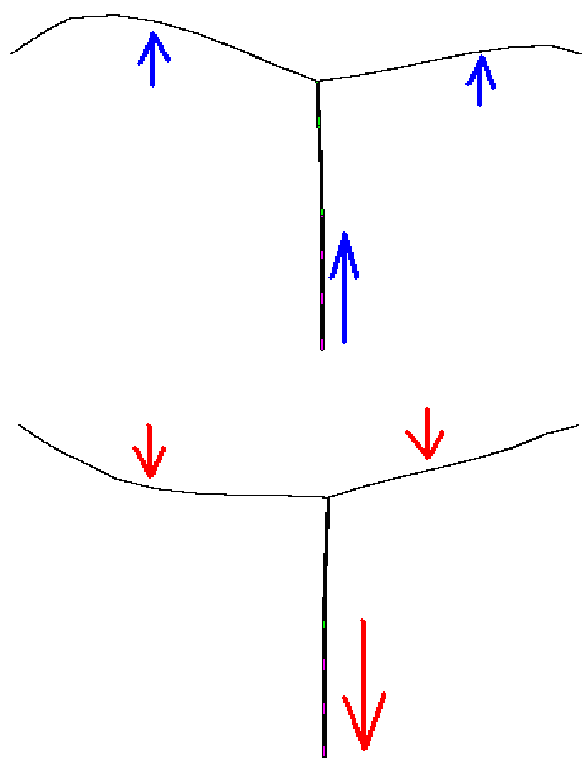


Figure B1: V shape transducer with a flat suspension at rest.



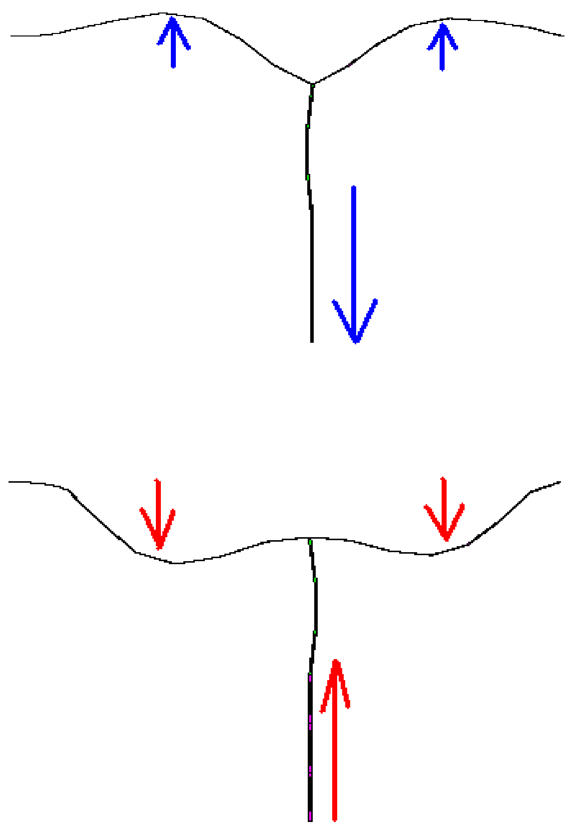


Figure B2: Upper: Main target mode of a V shape transducer with a flat suspension. Lower: Second target mode of the moving assembly. Only right side is depicted for clarity.

APPENDIX C: MAIN AXISYMMETRIC MODES OF A CONE. ORTHOGONAL AXIAL MODES.

The main modes of a truncated cone are very well explained in ref [24]. A basic mode on a cone is the breathing mode. The easiest way to understand the cone breathing is to imagine the shape of the cone while it is heated and cooled. If the cone had been free supported, then after heating it would became a bigger cone, and after cooling a smaller one. This motion or mode it is called the extensional mode, and it is very coherent. The extensional mode of a cone, which has attached a coil with its former, can be seen in reference [2]. The practical breathing of a cone it is commonly hidden by other circumstances, which are the presence of the coil in the moving assembly and due to the non neutral behavior of the suspensions (radial forces on suspensions). The cone breathing mode may be a target mode on a wide band speaker. As it is explained on [2], this mode usually appears at the upper region of the frequency response of the transducer.

In moving assembly dynamics there are other modes which are important and are target as well because the high contribution to the frequency response. One type of these basic modes is the called axial mode [2]. These modes may appear clear (evident) when analyzing a moving assembly, and in practice, are due to the cone's inertias and compliances and by the coil and former inertias and compliances as well. In some cases the suspension's dynamics hide these important modes, and might become less evident.

In practice, the axial modes include not only the cone but the coil as well; we simplify here the analysis to the single cone because theoretically it gives some light in respect to the target modes. Figure C1 depicts the mode-shapes of a cone for these two axial modes. The one on top has the main stiffness on the cone neck, and it is basically caused by the split of the moving assembly in two masses divided by the neck. The one on the bottom represent the split of the moving assembly mass in two masses due to the structurally weak cone rim. This particular mode can be excited if the neighbor suspension restrains substantially, or if this suspension is free to rotate at any circular contour as a *swivel joint* (this topic will be presented in future work). These two axial modes are orthogonal in respect to the system mass matrix. This point can be easily verified just applying the orthogonality relationship to the data delivered from the finite element solver for these two axial modes.

Observe that the generalized mass definition given before represents the amount of similitude that one particular mode has in respect to the system (the mass). Instead, the orthogonality condition, which is: $\Phi_i^T M \Phi_j = 0$, indicates the two modes i and j have nothing in common in respect to the mass of the system, which is the truncated cone itself. Notice that the generalized mass has been useful for finding relevant or target modes on this paper; and the orthogonality condition is useful to separate modes (by classes) in a cluster of them. See reference [25] for details of this topic. Observe that the practical sense of the "mass matrix" it is just the "system under investigation". Mass matrix M must not be considered as a weighting function, as some books describe [26]; because in dynamics, the mass matrix it is the system itself.

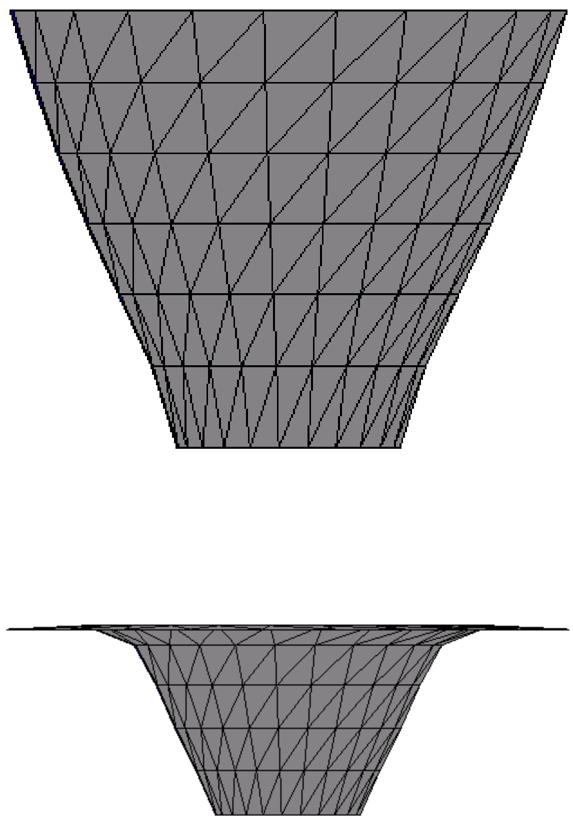


Figure C1: Upper: Mode 73 (1143 Hz) of a truncated cone restrained by the neck. Observe the main longitudinal motion due to in plane deformations. Lower: Mode 116 (1621 Hz). Observe the main radial motion due to cone rim bending.

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